Detrended Multifractalanalysis of Temperature and Precipitation Records

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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By using a hierarchy of methods, that can systematically eliminate trends, we analyze long-range correlations in temperature and precipitation records of more than hundred meteorological stations, spread around the globe. We find that in contrast to temperature records the precipitation records do not exhibit universal long-range correlations. Most of them are already uncorrelated on scales above one month. In the case of temperature records we find significant differences between stations on islands and on continents. Precipitation records exhibit stronger multifractal behavior than temperature records.

In general a **time series** of length L has the form:

$$x_i = z_i + m_i + s_i, \quad i = 1, \ldots, L$$

$s_i = s_{i+nT}$ = seasonal component
m_i = trend component
z_i = stochastic component (fluctuations)

Correlations between fluctuations of two values, separated by s time steps, of a time series are assigned by the **auto correlation function**

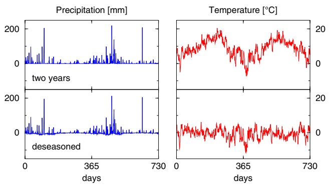
$$C(s) = \frac{1}{L-s} \sum_{i=1}^{L-s} z_i \, z_{i+s}$$

In the case of long-range correlations $C(s)$ decays like a power law:

$$C(s) \sim s^{-\gamma}, \quad 0 < \gamma < 1$$

Non-stationarities, like trends, and the finite length of the time series inhibit the determination of the asymptotic behavior of $C(s)$ and its **correlation exponent** γ [1].

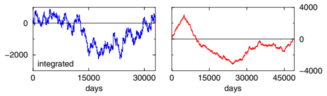
In order to analyze the correlations, trends and seasons must be eliminated. We determine the seasonal component s_i by calculating a mean value of x_i for each of the 365 days of the calendar. Unregularities due to seasons like pronounced droughts or rainy seasons cannot be removed completely. For eliminating the trends we use the **Detrended Fluctuation Analysis** (DFA).



Multifractal - Detrended Fluctuation Analysis (MF-DFA)

1. Cumulating the deseasoned series to a profile:

$$y_i = \sum_{k=1}^{i} \left(x_k - \langle x_k \rangle \right), \quad i = 1, \ldots, L.$$

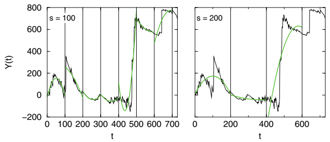


2. Dividing the profile into $N = L / s$ non-overlapping windows of size s, starting from both ends, gives $2N$ windows.

3. Using a polynomial approximation of degree n for each window $\nu = 1,\ldots,2N,$ the variance is determined by:

$$F_n^2(s,\nu) = \frac{1}{s} \sum_{i=1}^{s} \left(y_{(\nu-1)s+i} - p_{n,\nu}(i) \right)^2.$$

This procedure removes polynomial trends up to the order n from the profile [2].



4. Determining the qth moments (which according to the sign of q for large absolute values of q weighten small or big fluctuations [3])

$$F_n(q,s) = \left[\frac{1}{2N} \sum_{\nu=1}^{2N} \left(F^2(s,\nu) \right)^{q/2} \right]^{1/q}$$

and varying the window size s until s_{max}, as well as the q-values. When $F_n(q,s)$ follows

$$F_n(q,s) \sim s^{h(q)},$$

$h(q)$ and the classical **multifractal scaling-exponent** $\tau(q)$ in the standard multifractal analysis are related by:

$$\tau(q) = q\,h(q) - 1$$

[More details concerning the **Hölder**-Exponent and **singularity spectrum**: see poster of Stephan Zschiegner.]

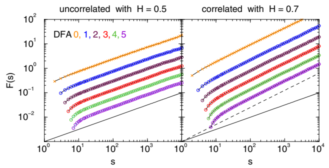
For $h(q) = const.$ we have a monofractal. For $q = 2$ the method corresponds to the usual DFA. In the presence of long-term correlations $h(2) = H$ is related to γ by

$$H = 1 - \gamma/2.$$

5. Measuring of the exponents $h(q)$ (including H) as the slopes of the functions $F_n(q,s)$ of **MF-DFA** in a log-log plot.

Examples of artificial data:

A) Second moments (q = 2) of uncorrelated and correlated random numbers for different orders n of approximation (DFA):

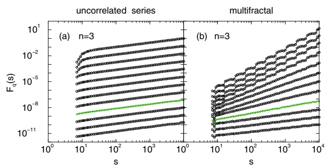


B) Multifractal analysis of uncorrelated random numbers and binomial multifractal systems, where

$$x_i = a^{n(i-1)} (1-a)^{n_{max} - n(i-1)},$$

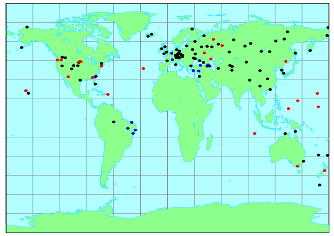
with $0.5 < a < 1$, $i = 1,\ldots, 2^{n_{max}}$ and $n(i)$ the number of 1-digits in the binary representation of the index i.

MF-DFA with fixed order $n = 3$ and q-values:
-10 ,-6 ,-4 ,-2, -1, -0.2, +0.2, +1, +2, +4, +6, +10



For the binomial multifractal system the multifractal scaling-exponent is

$$\tau(q) = -\frac{\ln(a^q + (1-a)^q)}{\ln(2)}$$

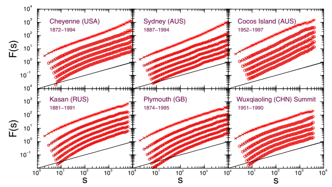


Analysis of climatological records

By applying **DFA** and **MF-DFA**, we analyzed temperature and precipitation records of **more than 100 stations**, in order to characterize long-range correlations and multifractality.

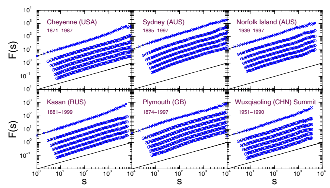
A) Long-Range Correlations:

Temperature records:



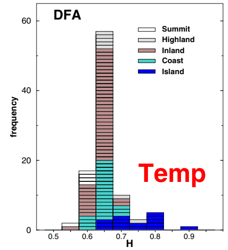
While in records of continental stations or stations close to the sea, a fluctuation exponent H of about 0.65 is found, small islands exhibit a much larger (approx. 0.80) and stations on summits a smaller (approx. 0.58) value. In any case long-range correlations are found in temperature records.

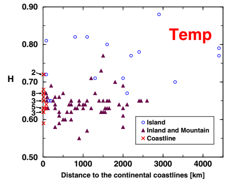
Precipitation records:

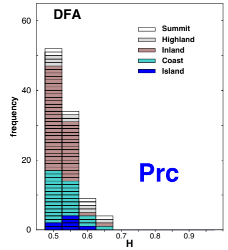


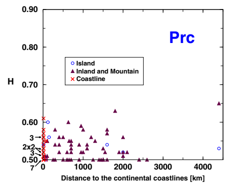
No pronounced long-range correlations are found in precipitation records. Often distinct short-range correlations occure.

With a geografical classification we achieved the following histograms. Further the dependance on the distance to the coastline is shown:



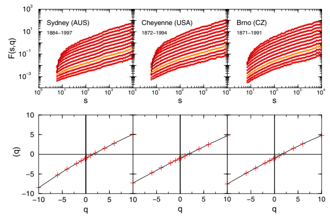






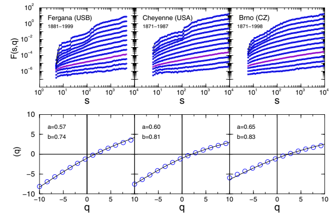
B) Multifractality:

Temperature records:



For temperature records **MF-DFA** exhibits only weakly different exponents $h(q)$. In the representation of $\tau(q)$ a slight bend is found, thus almost no multifractality.

Precipitation records:



In the $\tau(q)$-plot the precipitation records show a clear bend, which indicates pronounced multifractality.

In order to have a direct measure of the strength of multifractality, for precipitation records we use a non-linear curve fitting according to an extended binomial model

$$\tau(q) = -\frac{\ln(a^q + b^q)}{\ln(2)}$$

with two fitting parameters a and b.

Then the width of the singularity spektrum $f(\alpha)$ is given by [4]

$$\Delta\alpha = h(-\infty) - h(\infty) = \frac{\ln b - \ln a}{\ln 2}$$

Summary

Temperatures:

- "universal" long-range correlations with a fluctuation exponent H of about 0.65
- exceptions: islands have bigger, summits smaller fluctuation exponents H
- barely multifractality

Precipitations:

- mostly no universal long-range correlations
- often pronounced multifractality, especially in tropical and subtropical regions

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