Network properties of meteorological stations connected by rank of phase synchronization or cross-correlation

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We study the phase synchronization and cross-correlation of precipitation records and determine for each pair of records the values for best phase synchronization and cross-correlation by varying the time lag s. We consider 317 stations located in the river Elbe-basin as nodes, which we connect one by one according to their rank ordered values of maximum phase synchronization or cross-correlation, after subtraction the trivial influence of distance. We analyze the statistical properties of the emerging networks and find for both remarkable differences with the random networks, in particular when cluster sizes and degree distributions are considered.

Analysis of phase synchronization

The condition for phase-locking of two records $x_t^{(1)}$ and $x_t^{(2)}$ is given by:

$$|n\phi_t^{(1)} - m\phi_t^{(2)} - \delta| < \text{const.}$$

In a more general perception, phase synchronization occurs when their phase-difference is statistically agglomerated. In order to verify it, we use various steps:

1. Calculate the instantaneous phases via Hilbert-Transform $y_t^{(1,2)} = \frac{1}{\pi}\mathrm{CP}\int_{-\infty}^{\infty}\frac{x_t^{(1,2)}}{t-\tau}d\tau$ and $\tan\phi_t^{(1,2)} = y_t^{(1,2)}/x_t^{(1,2)}$ for normalized $x_t^{(1,2)}$.
2. Unwrap the phases by incrementing (or decrementing) by 2π each time a turn is completed.
3. Determine the phase-difference $\varphi_t^{n,m} = n\phi^{(1)} - m\phi^{(2)}$ and the cyclic phase-difference $\psi_t^{n,m} = \varphi_t^{n,m} \mathrm{mod} 2\pi$.
4. Quantify Agglomeration of $\psi_t^{n,m}$, e.g. histogram and Shannon-Entropy ($\rightarrow$ synchronization index ρ).
5. Repeat steps with shifted recods by s units [1].

In contrast to cross-correlation analysis, we expect complementary results from phase synchronization analysis, since only the phases are considered and the amplitudes are neglected.

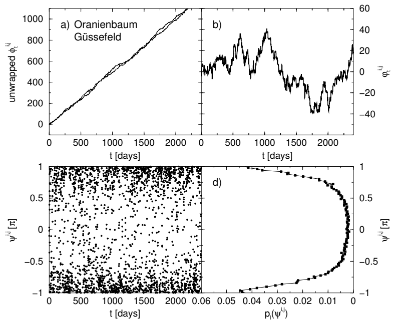

Figure 1

In Figure 1a) the unwrapped phases of two precipitation records are shown and in b) their phase-difference ($n : m = 1 : 1$). The cyclic phase-difference is given by b) and a histogram of $\psi_t^{n,m}$ is plotted in d). Phase-differences around $\pm\pi$ are much more frequent.

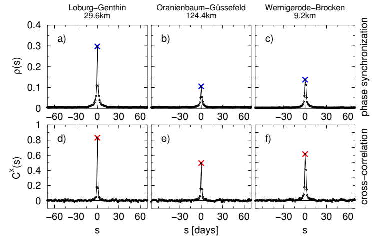

Figure 2

Figure 2 depicts three examples of phase synchronization ($\rho(s)$, upper row) and cross-correlation ($C^\times(s)$, lower row) results for precipitation records. Here at $s = 0$ strong phase synchronization and cross-correlation is found, which vanish for $|s| \gg 0$.

Application to precipitation records

We consider 317 daily precipitation records of the period 1951-2000 (18250 days each) of stations located in the german part of the river Elbe-basin, shown in Fig. 3.

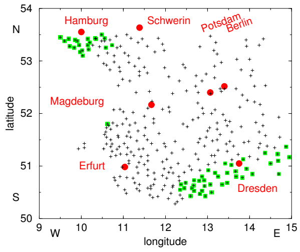

Figure 3

We calculate $\rho(s)$ and $C^\times(s)$ for each with each record and obtain $317(317-1)/2 = 50086$ combinations. For all of them we verify the appearance of a peak (Fig. 2) and store maximum values $s_{\max}$ and $\rho_{\max}$, $C^\times_{\max}$. In most of the cases $s_{\max} = 0$ is found and only 238 (0.4%) for $\rho(s)$ and 129 (0.3%) for $C^\times(s)$ have a maximum at $s_{\max} = \pm 1$ day (colored in Fig. 4).

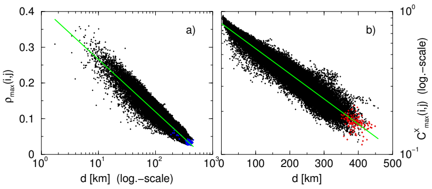

Figure 4

In Figure 4 the results are plotted against the distance between the sites. In a) it can be seen that the strength of phase synchronization decays logarithmically while in b) the strength of cross-correlations decays exponentially. We perform least square fits and obtain $\rho(d) = -0.06\ln(d) + 0.41$ and $C^\times(d) = 0.83\exp(-0.004d)$ which leads to an average maximum range about 650km for phase synchronization and an infinity range for cross-correlation.

Network construction and properties

Next we understand the sites as nodes of a network which we consequently link according to the strength of cross-correlation [2,3] or phase synchronization after subtraction of the trivial influence of the distance between the sites. We start from the empty set of nodes and add the links one by one according to the rank ordered values of $\rho_{\max} - \rho(d)$ or $C^\times_{\max} - C^\times(d)$ respectively.

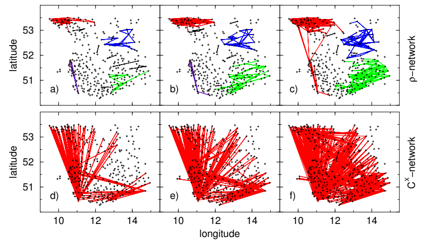

Figure 5

Figure 5 shows the network(s) after 128, 256, or 512 links added. In the case of phase synchronization distinct clusters are found, while in the case of cross-correlation one large cluster with many long-distance connections emerges.

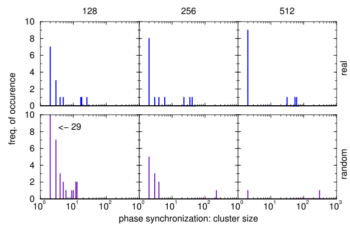

Figure 6

Figure 6 shows histograms of cluster-sizes for the corresponding cases. For a random configuration very soon a large cluster appears.

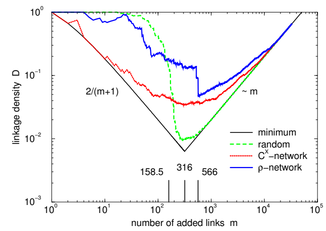

Figure 7

In Figure 7 we plot the linkage density

$$D = \frac{\text{number of links}}{\text{possible links}} = m \Big/ \left(\frac{1}{2} \sum_{\text{cluster } k} n_k(n_k - 1) \right)$$

with $D_{\min} = 2/(m+1)$, $2 \le n \le 317$, for phase synchronization, cross-correlation and a random configuration. The ρ-network is highly connected and exhibits a collapse when 566 links are added. In comparison to the random configuration, the $C^\times$-network initially exhibits very low density, while it remains higher connected at the end.

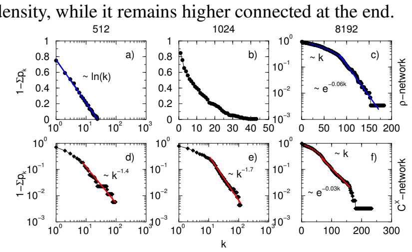

Figure 8

The degree distribution p_k for the networks at different stages is given by Fig. 8. For the ρ-network it follows $1/x$ at the beginning and later an exponential decay. In the case of the $C^\times$-network its tail follows a power-law and later also an exponential decay.

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References:

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