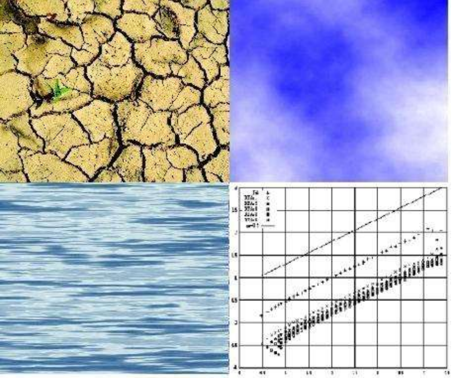


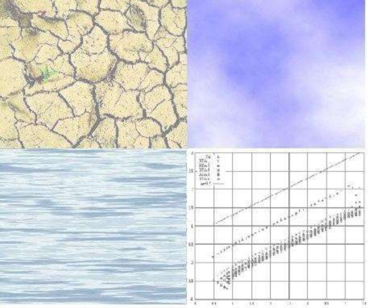
R&D Research Project: Scaling analysis of hydrometeorological time series data

Precipitation and river runoff records: Long-term persistence and multifractality

Diego Rybski

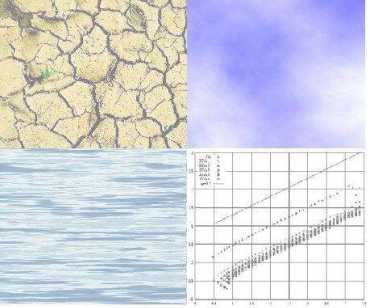
J.W. Kantelhardt, E. Koscielny-Bunde, P. Braun, A. Bunde, S. Havlin

Potsdam, 13.12.2005



Outline

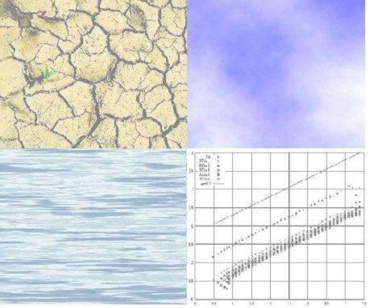
1. Introduction
2. Method and Data
3. Correlation Behavior
4. Multifractal Characterization
5. Summary and Conclusion



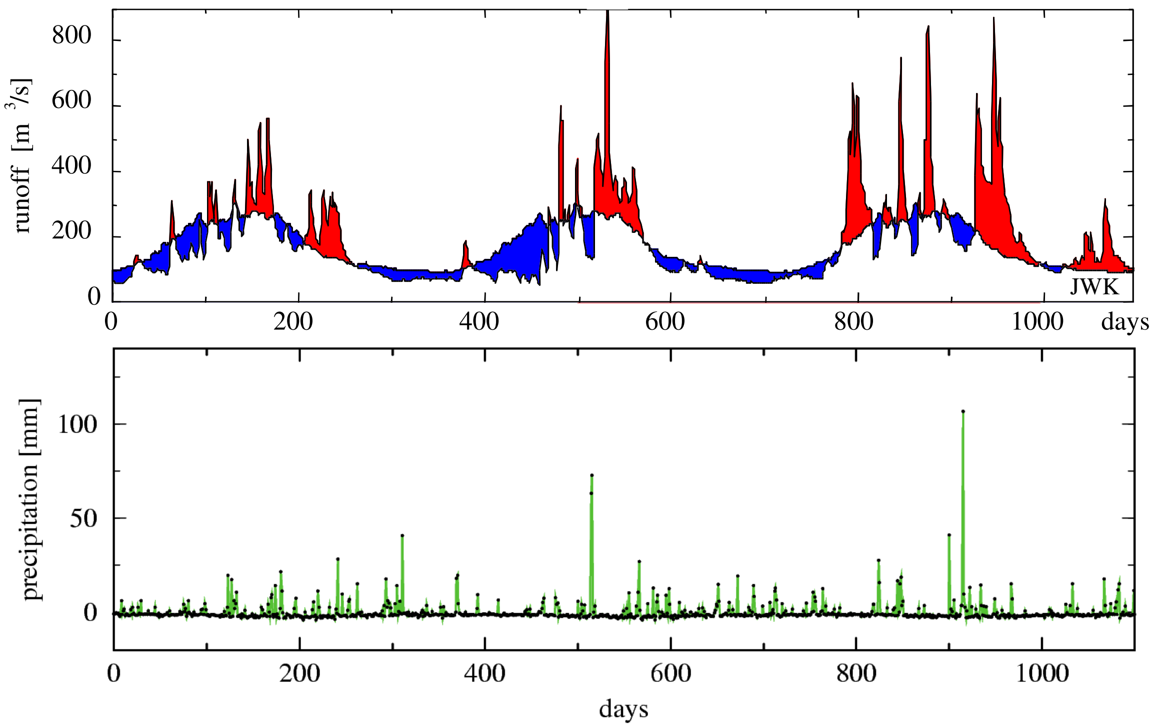
Introduction

Motivation:

- new methods enable precise characterization of temporal scaling
- on large scales, also in the presence of non-stationarities
- precipitation discussed controversially
- precipitation and runoff records require multifractal description



Long-term persistence



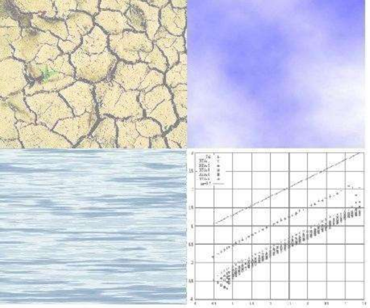
auto-correlation function:

$$C(s) = \frac{\langle \tau_i \tau_{i+s} \rangle}{\langle \tau_i^2 \rangle}$$

$$= \frac{1}{(N-s)\langle \tau_i^2 \rangle} \sum_{i=1}^{N-s} \tau_i \tau_{i+s}$$

<u>correlation-time:</u> $s_\times = \int_0^\infty C(s) \mathrm{d}s$

- infinite (long-term), e.g. power-law
- finite (short-term), e.g. exp. decay



Method:
Detrended Fluctuation Analysis

1. deseasoning:

$$\tau_i = x_i - \langle x_i \rangle$$

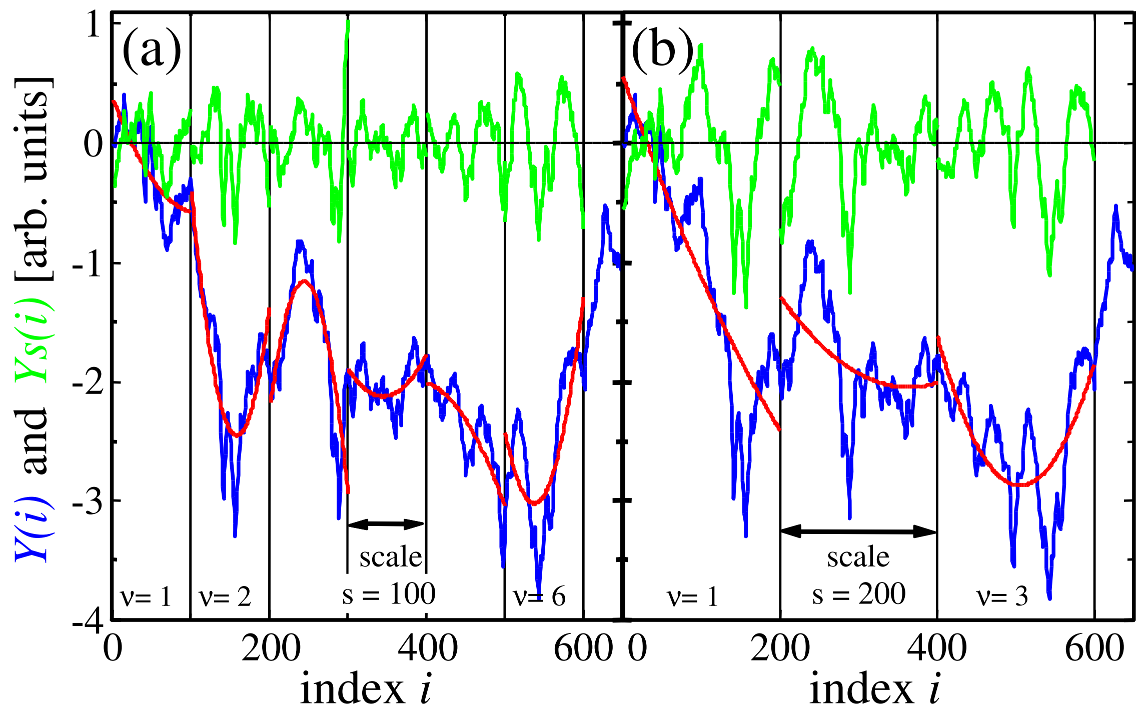
2. create profile:

$$Y(k) = \sum_{i=1}^{k} \tau_i$$

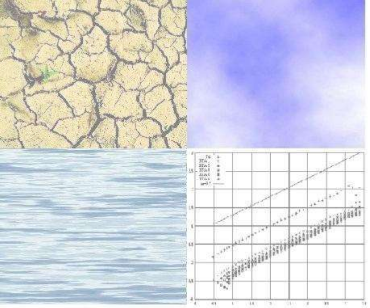
3. separate into non-overlapping windows of size s

4. polynomial trend in each window and difference:

$$Y_s(i) := Y(i) - p_\nu(i)$$



Kantelhardt, J.W. et al. 2001



Method:
Detrended Fluctuation Analysis

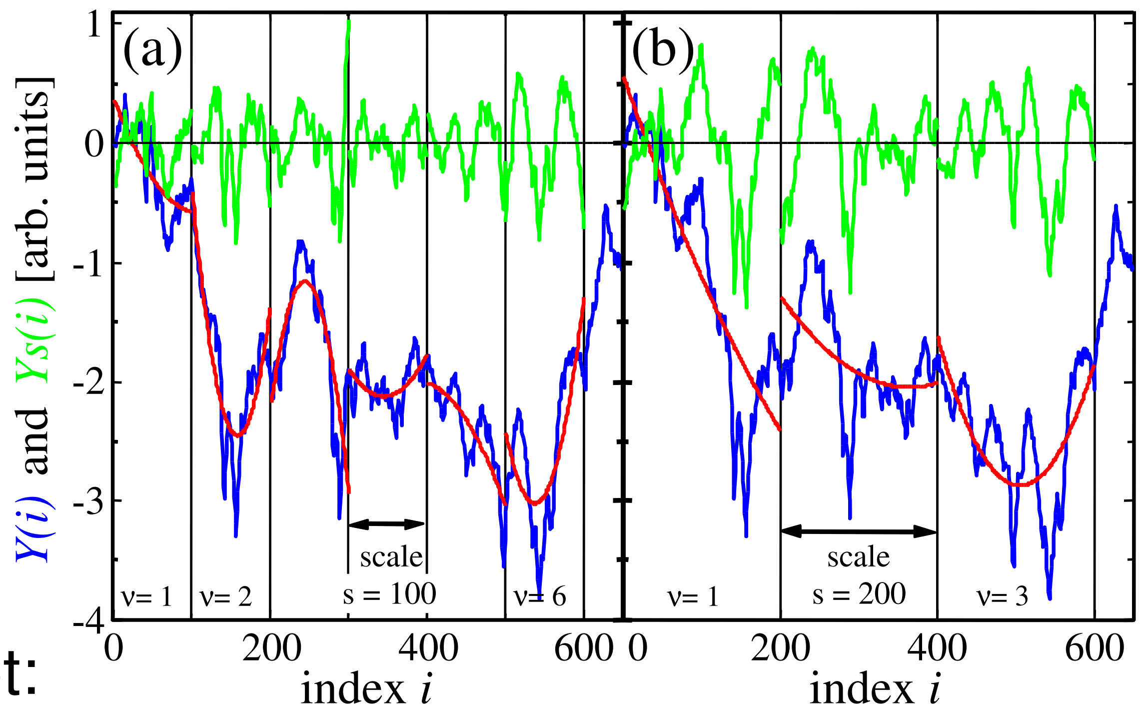
5. variance in each window:

$$F_s^2(\nu) = \langle Y_s^2(i) \rangle$$

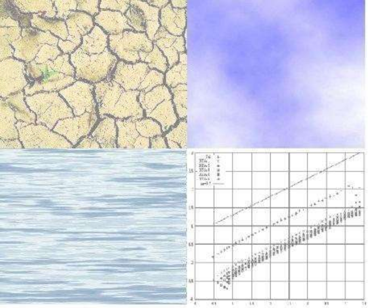
$$= \frac{1}{s} \sum_{i=1}^{s} Y_s^2((v-1)s+i)$$

6. average of all windows, square-root:

$$F(s) = \left(\frac{1}{2N_s} \sum_{\nu=1}^{2N_s} F_s^2(\nu) \right)^{\frac{1}{2}}$$



Kantelhardt, J.W. et al. 2001



Method:
Detrended Fluctuation Analysis

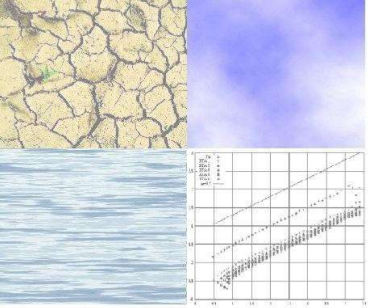
Properties:

- power-law with fluctuation-exponent: $F^{(n)}(s) \sim s^{\alpha}$
- related to correlation-exponent: $\alpha = 1 - \gamma/2 \quad C(s) \sim s^{-\gamma}$
- different cases:

$\alpha \simeq 0{,}5$	uncorrelated
$0{,}5 < \alpha < 1{,}0$	long-term correlated
$1{,}0 < \alpha$	non-stationary

Detrending:

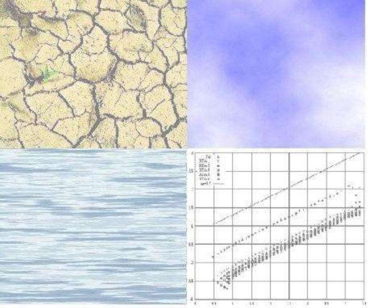
- polynomial order n: linear (DFA1), parabolic (DFA2), ... DFAn
- DFA2: removes 2nd-order trends in profile
  => linear trends in original sequence



Data

99 precipitation records:

- Europe (40), Asia (34), North-America (15), South-America (5), Australia (5)
- latitude: 52.6 S ... 71.3 N, average 41 N
- elevation: sea level ... 3650 m, average 400m
- tropical (12), sub-tropical (24), warm (44), cold (13), polar (6)
- arid (1), semi-arid (20), semi-humid (59), humid (19)
- lengths: 34y ... 189y, average 86y (12,000-69,000)

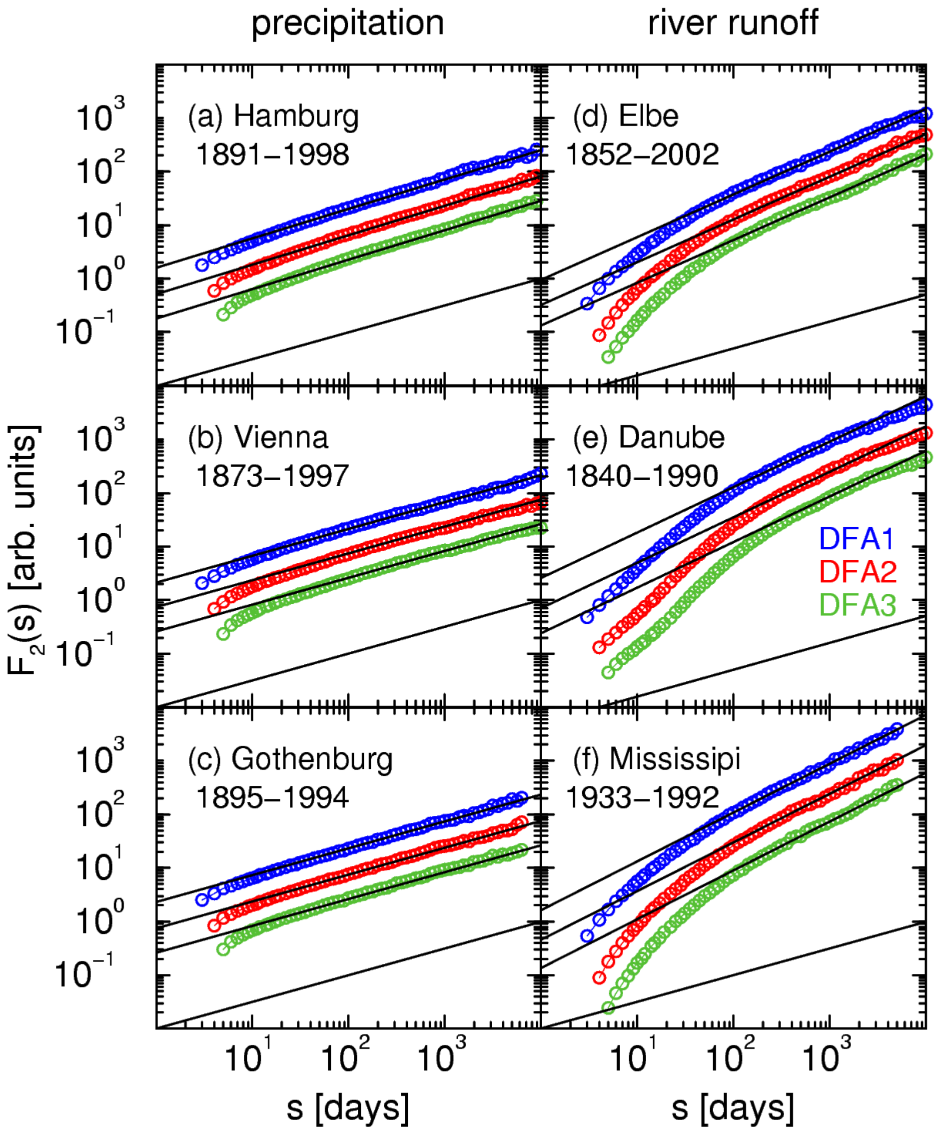


Correlation Behavior

0.55 +/- 0.03

0.50 +/- 0.03

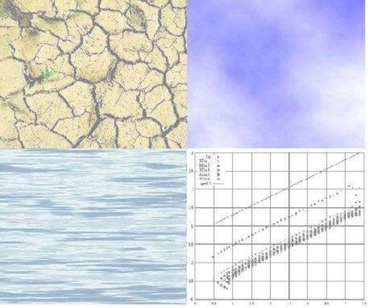
0.50 +/- 0.03



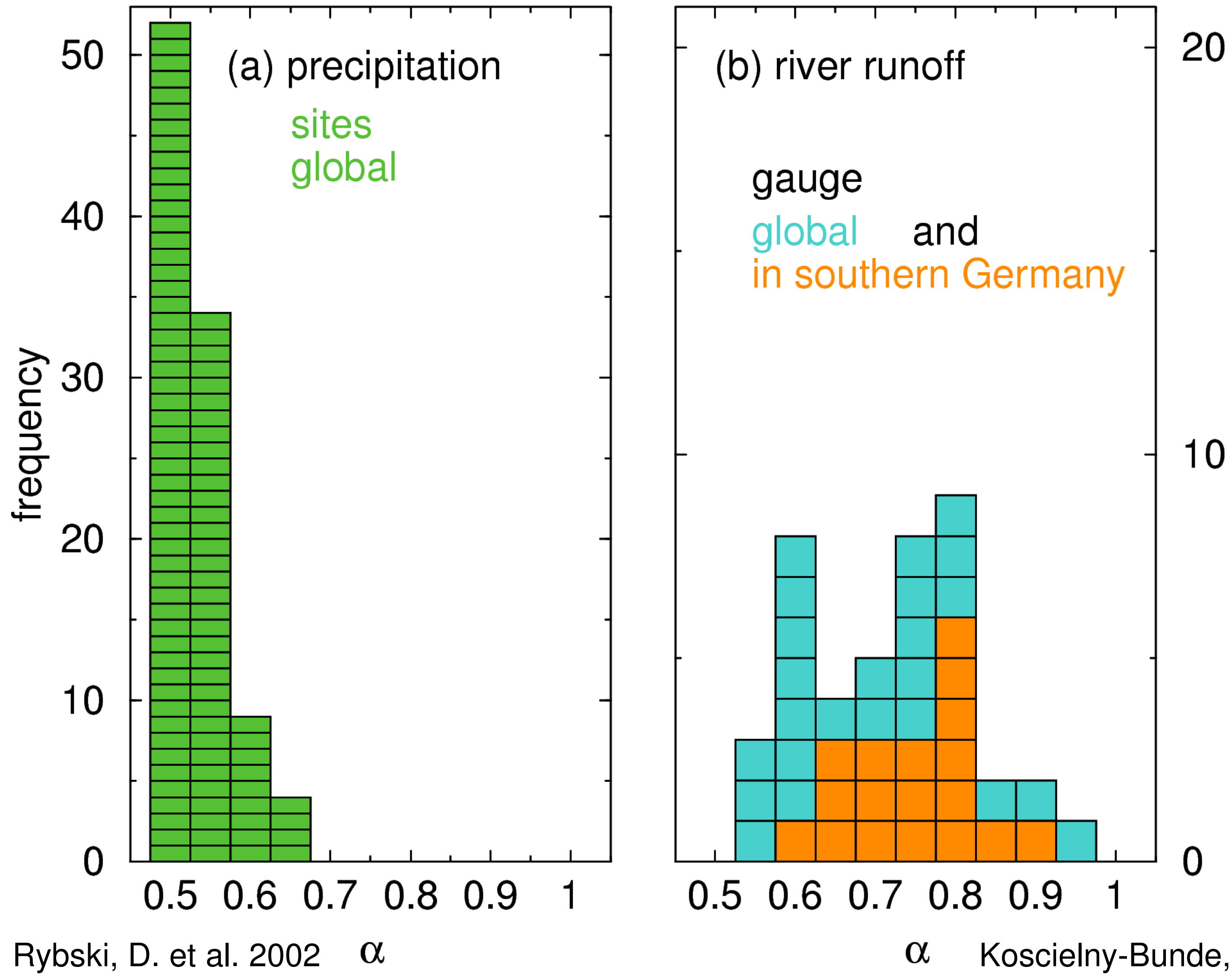
0.80 +/- 0.03

0.85 +/- 0.03

0.91 +/- 0.03

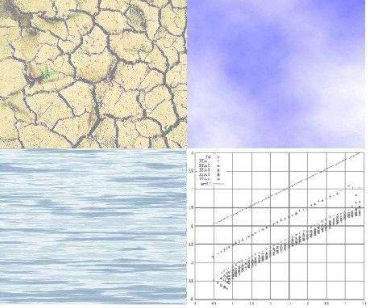


Correlation Behavior



Rybski, D. et al. 2002

Koscielny-Bunde, E. et al. 2005



Multifractal - Detrended Fluctuation Analysis

6. average of all windows, q th root:

$$\text{DFA:} \qquad F(s) = \left(\frac{1}{2N_s} \sum_{\nu=1}^{2N_s} F_s^2(\nu) \right)^{\frac{1}{2}}$$

$$\text{MF-DFA:} \qquad F_q(s) = \left(\frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F_s^2(\nu)]^{\frac{q}{2}} \right)^{\frac{1}{q}}$$

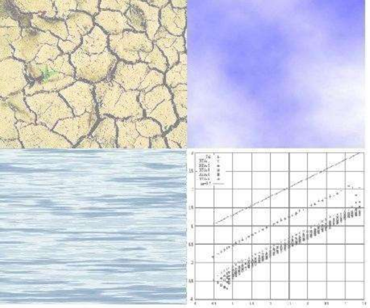
- multi-scaling, fluctuation-exponent depends on moment:

$$F_q(s) \sim s^{\alpha(q)}$$

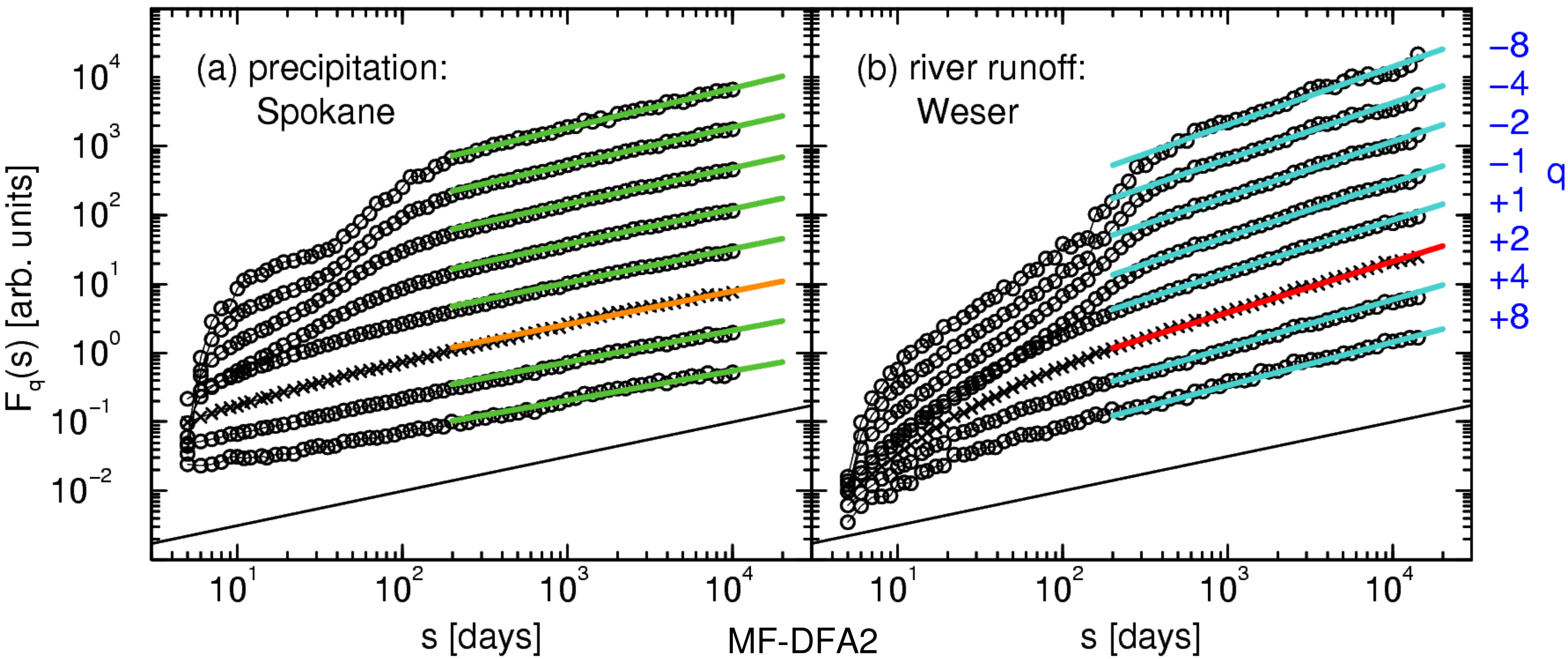
- positive moment q : windows with big fluctuations dominate
  negative moment q : windows with small fluct. dominate

- strength of multifractality:

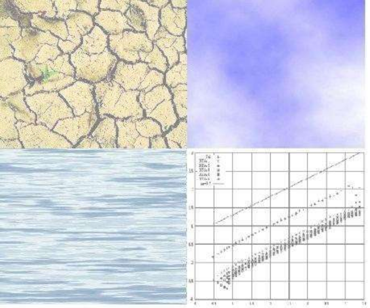
$$\begin{aligned} \Delta\alpha &= \alpha(q)^{\max} - \alpha(q)^{\min} \\ &= \alpha(q \to -\infty) - \alpha(q \to +\infty) \end{aligned}$$



Multifractal Characterization



generalized Hurst-exponent: $\alpha(q)$ (also denoted by $h(q)$)

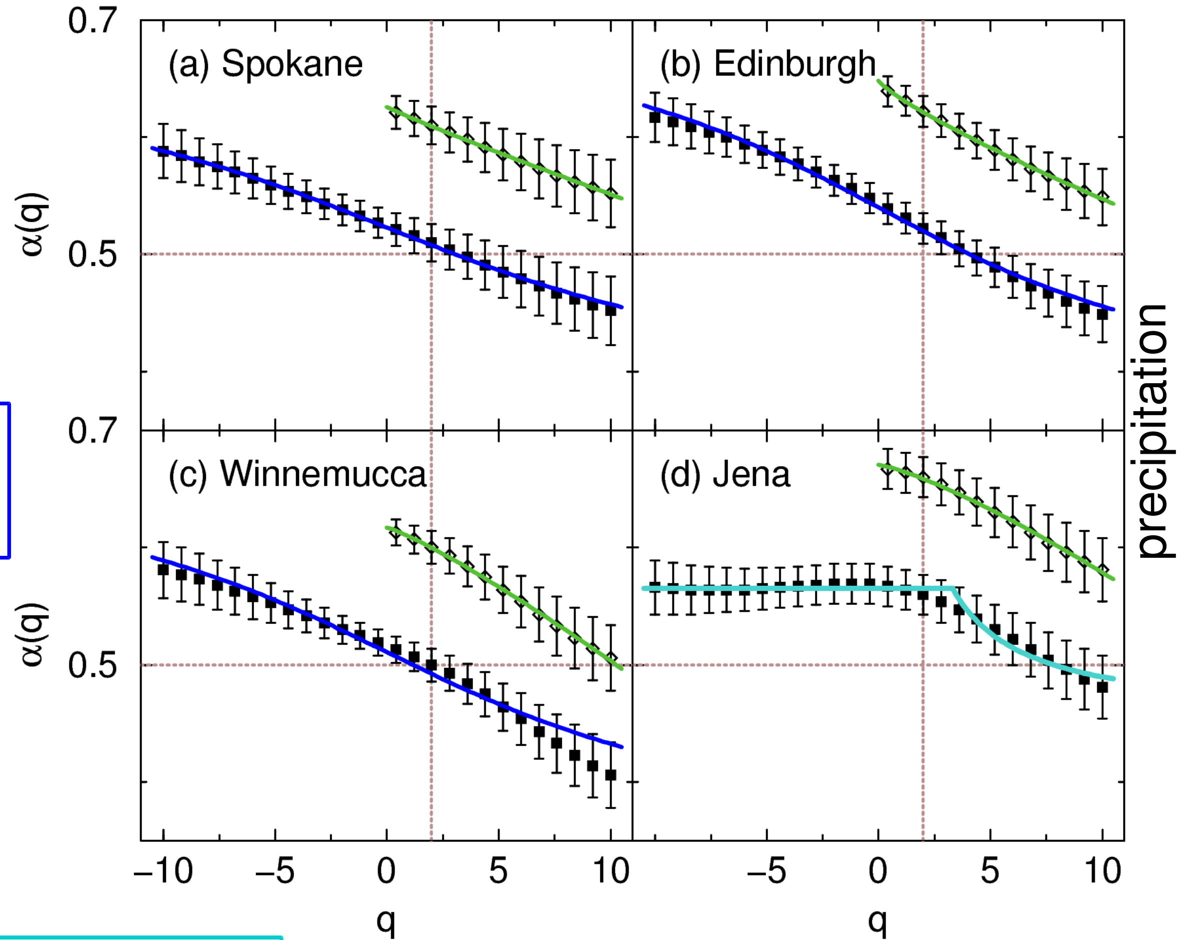


Multifractal Characterization

three operational models:

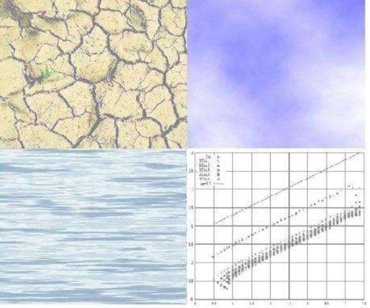
$$\alpha(q) = \frac{1}{q} - \frac{\ln(a^q + b^q)}{q \ln 2}$$

$$\Delta\alpha = \frac{\ln b - \ln a}{\ln 2}$$



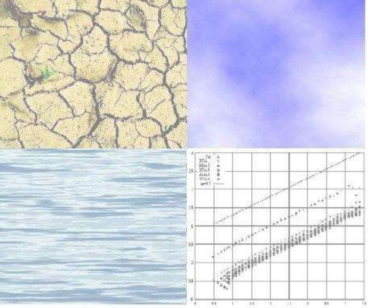
$$\alpha(q) = \begin{cases} \alpha_1 & q \le q_\times \\ q_\times(\alpha_1 - \alpha_2)\frac{1}{q} + \alpha_2 & q > q_\times \end{cases}$$

$$\alpha(q) = \aleph' + 1 - \frac{C_1}{a' - 1}\left(q^{a'-1} - 1\right)$$

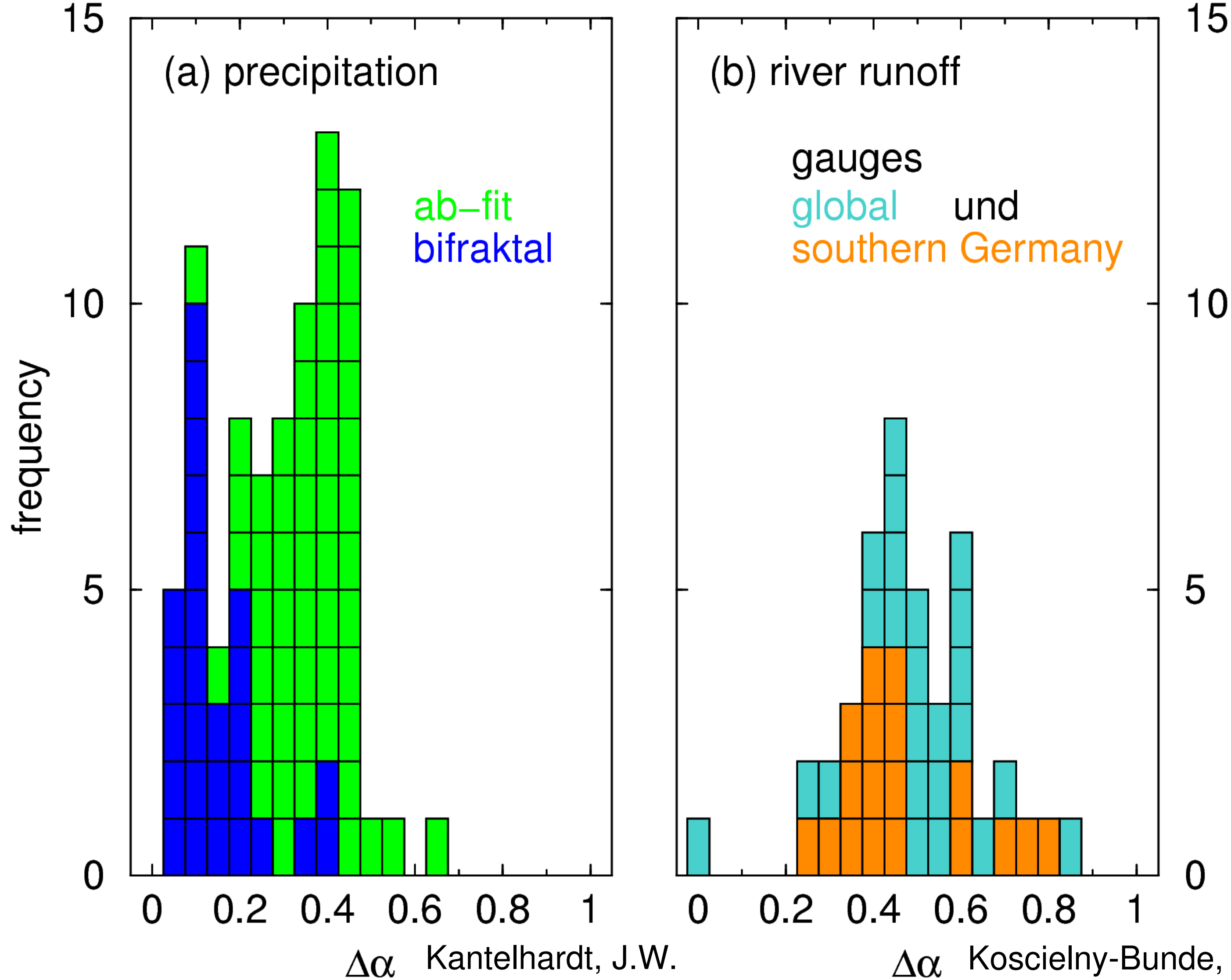


Multifractal Characterization

precipitation:	period	$\alpha(2)$	$\Delta\alpha$	a	b
Spokane	1881-1994	0.51	0.30	0.63	0.77
Edinburgh	1896-1997	0.52	0.34	0.61	0.77
Winnemucca	1877-1994	0.50	0.33	0.63	0.79
Jena	1827-2000	0.56	0.11	–	–
average (99 rec.)	86y	0.53	0.29	–	–
std. deviation	33y	0.04	0.14	–	–
runoff:					
Weser	1823-1993	0.76	0.43	0.50	0.68
Fraser	1913-1996	0.69	0.38	0.53	0.70
Susquehanna	1891-1986	0.58	0.48	0.55	0.77
Niger	1907-1985	0.60	0.62	0.51	0.78
average (42 rec.)	86y	0.72	0.49	–	–
std. deviation	27y	0.11	0.16	–	–

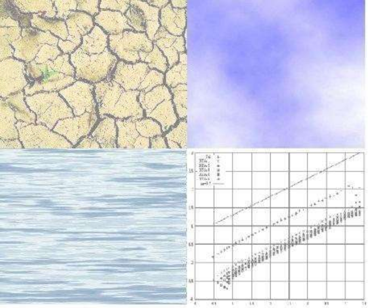


Multifractal Characterization



Kantelhardt, J.W. et al. 2006

Koscielny-Bunde, E. et al. 2005



Summary and Conclusion

- precipitation uncorrelated on large scales
- runoff records long-term correlated
- persistence of runoff due to storage processes
- multifractality (mf.) for both, precipitation and runoff
- runoff records exhibit stronger mf.
- modified binomial mf. model (**2 parameters**) for positive and negative moments
- many prec. records require different model (45%)
- bifractal approach (3 param.) fits in 27%
- Lovejoy-Schertzer approach (3 param.) fits always well for positive moments)

J.W. Kantelhardt, E. Koscielny-Bunde, D. Rybski, P. Braun, A. Bunde, S. Havlin, JGR Atmos., 2006.