

Auerbach, Lotka, and Zipf: pioneers of power-law city-size distributions [1] – Tab. I

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(Dated: November 2, 2023)

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Author	Year	Ref.	Trans.	Type	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	Comment
Pareto	1896	[2]		book	o											French
Auerbach	1913	[3]	[13]	paper		o										German, no reference at all
Lotka	1925	[4]	-	book	.	x	o									
Goodrich	1926	[5]	-	paper				o								no references at all
Saibante	1928	[6]		paper	x	x	x		o							Italian, first connection to Pareto 1896 [2]
Gibrat	1931	[7]		book	.				a	o						French, finds log-normal distribution
Christaller	1933	[8]	[14]	book		x				o						German
Singer	1936	[9]	-	paper	x		b		x		o					
Lösch	1940	[10]	[15]	book	x				x	x	x	o		c		German, mentions “Singer’s discovery”
Stewart	1947	[11]	-	paper	.	x	x		.			o		c		
Zipf	1949	[12]	-	book	.	x	x	x	x	.		.	o			mentions “Pareto school”

Table I: Citation matrix. \times , ‘proper’ referencing; \cdot , either mentioning or citing in a different context; \circ , diagonal elements; ^a, Saibante 1926 [16]. ^b, Goodrich 1928 [17]. ^c, Zipf 1941 [18]. Lösch’s book first appeared in 1940 but we consider the translation which is based on the 1944 2nd edition. We would like to note that we compiled this table with the best of our knowledge but cannot exclude that we miss a citation. “Ref.” stands for reference and “Trans.” for translation to English.

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