About an indirect determination of damage functions (SSC2010-135 P65)

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In order to estimate future damage caused by natural disasters, it is desirable to know the damage caused by single events. So called damage functions provide – for a natural disaster of certain magnitude – a specific damage value. However, in general, the functional form of such damage functions is unknown. We study the distributions of recorded damage values and deduce which damage functions lead to such distributions when the natural disasters obey Generalized Extreme Value statistics. We find broad damage distributions and investigate two possible functional forms to characterize the data. In the case of Gumbel distributed extreme events, a power-law distribution density with an exponent close to 2 (Zipf's law) implies an exponential damage function. Stretched exponential distribution densities imply power-law damage functions. In the case of Weibull (Fréchet) distributed extreme events we find correspondingly steeper (less steep) damage functions.

Motivation

One of the consequences which are expected to come along with global warming is an increased number of climate related disasters. The question raised by governments or insurances is, which costs certain regions, countries, or even the globalized economy are facing.
Natural disasters, such as floods or storms, represent extreme events. Independent of the problem on how to project future extreme events, one is interested in which damage can typically be expected from an extreme event of certain magnitude. One approach to tackle this question is to separate the statistics of extreme events from the damage caused by them. The former can be obtained from measurements, such as water level records, but the latter needs to be estimated by some empirical studies or by theoretical considerations.
Accordingly, so called damage functions provide a monetary value as a function of the magnitude of an event, such as the maximum level of a flood. One distinguishes damage functions on a microscopic scale from those on a macroscopic scale. The former describes the typical costs of damages to single assets, such as residential buildings, and the latter describes costs of damages to larger areas, such as a city. This macroscopic damage function represents a composition of information on asset values, their location, and their vulnerability, see e.g. [1]. If not specified we refer to macroscopic damage functions, in what follows.
The question we address is, which functional form a damage function must follow so that the distribution of extreme events transforms to the distribution of damages. Relating these two distributions we obtain macroscopic damage functions. Therefore we analyze flood data assembled by CRED [2] and find broad damage distributions. In order to characterize them, we elaborate two functional forms.

Extreme Events

As it is known analytically, extreme events – defined as maximum values of samples of fixed size (such as "block maxima" of time series) – follow distributions which converge towards Generalized Extreme Value (GEV) distributions (for sufficiently large samples). Thus, GEV distributions are fitted to data of extreme events in order to estimate annualities and future occurrences. For a more detailed presentation of extreme value assessment and applications we refer to [3].
The GEV distributions, expressing the probability that the maximum of a sample is beneath the value s, are given by:

$$P_{(s)}^{\mathrm{GEV}} = \begin{cases} \exp\left[-\left(1+\xi\frac{s-\nu}{\gamma}\right)^{-\frac{1}{\xi}}\right] & \text{for } \xi \neq 0 \\ \exp\left[-e^{-\frac{s-\nu}{\gamma}}\right] & \text{for } \xi = 0\,. \end{cases} \quad (1)$$

They are defined on $\left\{s : 1+\xi\frac{s-\nu}{\gamma} > 0\right\}$ and have a location parameter, $\nu \in \mathbb{R}$, a scale parameter, $\gamma \in \mathbb{R}^+$, as well as a shape parameter, $\xi \in \mathbb{R}$. According to the shape, one distinguishes three cases:

1. the Gumbel distribution ($\xi = 0$),
2. the heavy-tailed Fréchet distribution ($\xi > 0$), and
3. the bounded-tailed Weibull distribution ($\xi < 0$).

Empirical damage distributions

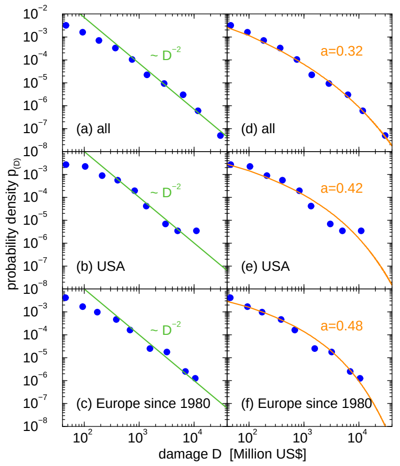

Figure 1

We consider the EM-DAT database [2] collected by the Centre for Research on Epidemiology of Disasters (CRED) in version v12.07 as created on Oct-28-2009. The information listed for each event entry consists of: start, end, country, location, type of disaster, sub-type, name, number of people killed, number of people affected, an estimated damage, and an ID. For the years 1950-2008 we extract the information on floods, which include general floods, flash floods, as well as storm surges respectively coastal floods, and obtain 3469 entries worldwide, while for 1225 entries an estimated damage in units of Million US-Dollars is available.
In Figure 1(a) we show the estimated probability densities, $\widetilde{p}(D)$, for all flood damage values of the database. However, since one may argue that the result could be biased by regional differences, in Fig. 1(b) we show $\widetilde{p}(D)$ for only those floods that occurred in the USA. Thus, we can neglect influences due to different economic power of different countries. In order to reduce possible trends in the data, we also exclude floods before 1980 and plot the distribution density for Europe in Fig. 1(c). In any case, we observe broad distributions with damages reaching the order of 10 Billion US-Dollars. We would also like to note that we obtain similar results for the number of killed or affected people as well as for other natural disasters.
In the simplest approach, the tail of the probability densities can be described with a functional form involving one parameter, namely a power-law according to

$$\widetilde{p}_{(D)} \sim D^{-\alpha}\,, \quad (2)$$

where we find $\alpha \approx 2$ [Fig. 1(a-c)]. Such a size distribution is also known as Zipf's law and is found in many different fields, such as word usage, city sizes, firm sizes, wealth, intensity of solar flares, ... For an overview we refer to [4] and references therein. Minor deviations from Eq. (2) for floods with small damage, could be due to the fact that small damages are more likely to be missing in the database.

Determining damage functions

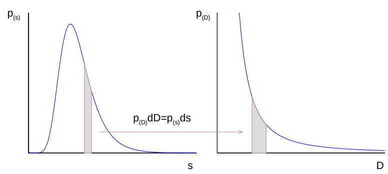

Figure 2

If we assume a unique relation between the magnitude of an extreme event (such as high river or sea levels), s, and the damage, $D_{(s)}$, that is caused by it, we can take advantage of Eq. (1) and (2) and relate them. Therefore, we write the probability as an integral over the density and substitute the magnitude with the damage $\left(s \to D = D_{(s)}\right)$:

$$\int_{s_1}^{s_2} p_{(s)}\mathrm{d}s = \int_{D_{(s_1)}}^{D_{(s_2)}} p_{(s_{(D)})}\frac{\mathrm{d}s}{\mathrm{d}D}\mathrm{d}D = \int_{D_{(s_1)}}^{D_{(s_2)}} \widetilde{p}_{(D)}\mathrm{d}D\,. \quad (3)$$

Here, $p_{(s)}$ and $\widetilde{p}_{(D)}$ are the probability density of the extreme event and the damage, respectively. Furthermore, the density transformation $\frac{\widetilde{p}_{(D)}}{p_{(s)}} = \frac{\mathrm{d}s}{\mathrm{d}D}$ was used (see illustrative Fig. 2). Next, choosing $s_1 = s_0$ (the lower bound of the GEV-distribution) and $s_2 = s$, we obtain the equation

$$P_{(s)} = \int_{D_{(s_0)}}^{D_{(s)}} \widetilde{p}_{(D)}\mathrm{d}D\,, \quad (4)$$

which holds for any (reasonable) damage distribution density $\widetilde{p}_{(D)}$.
Using Eq. (2), i.e. the probability density $\widetilde{p}_{(D)} = AD^{-\alpha}$ $(\alpha > 1)$ as found in Fig. 1(a-c), we write

$$P_{(s)} = \int_{D_{(s_0)}}^{D_{(s)}} AD^{-\alpha}\mathrm{d}D = 1 - D_{(s_0)}^{\alpha-1}D_{(s)}^{1-\alpha}\,, \quad (5)$$

where $A = (\alpha-1)D_{(s_0)}^{\alpha-1}$ such that $\widetilde{p}_{(D)}$ is normalized in $D \in \left[D_{(s_0)}, D_{(s\to\infty)}\right)$. Solving the equation for $D_{(s)}$ we get

$$D_{(s)} = D_{(s_0)}\left(1 - P_{(s)}\right)^{\frac{1}{1-\alpha}}\,. \quad (6)$$

Finally, we insert the Gumbel distribution, i.e. Eq. (1) with $\xi = 0$, and obtain

$$D_{(s)} = D_{(s_0)}\left(1 - \exp\left[-e^{-\frac{s-\nu}{\gamma}}\right]\right)^{\frac{1}{1-\alpha}} \quad (7)$$

Asymptotically it increases exponentially, which becomes clear when the following approximation is employed.
The Generalized Pareto Distribution (GPD) is an approximation of the distribution function of the level s above the threshold s_T, under the condition that the maximum of the sample exceeds the threshold [3]. Then, the probability distribution of maxima that exceed the threshold is described by

$$P_{(s)}^{\mathrm{GPD}} = \begin{cases} 1 - \left(1+\xi\frac{s-s_T}{\widetilde{\gamma}}\right)^{-\frac{1}{\xi}} & \text{for } \xi \neq 0 \\ 1 - e^{-\frac{s-s_T}{\widetilde{\gamma}}} & \text{for } \xi = 0\,, \end{cases} \quad (8)$$

where $\widetilde{\gamma} = \gamma + \xi(s_T - \nu)$ and $s \in [s_T, \infty)$. Using the Gumbel case ($\xi = 0$) of the GPD instead of Eq. (1) in Eq. (6) we obtain that asymptotically the damage increases exponentially,

$$D_{(s)} = D_{(s_T)}e^{\frac{s-s_T}{\widetilde{\gamma}(\alpha-1)}}\,, \quad (9)$$

which holds for $s \in [s_T, \infty)$. Thus, if we assume power-law distributed damages as well as extreme events following the Gumbel distribution, then the damages caused by such natural disasters must (asymptotically) depend exponentially on the magnitude of the corresponding extreme events.

However, this exponential damage function is based on damages following power-law distributions [Fig. 1(a-c)]. Since a one parameter description might not be sufficient, we also elaborate a two parameter fit, namely a stretched exponential according to

$$\widetilde{p}(D) \sim \frac{a}{b}D^{a-1}e^{-\frac{D^a}{b}}\,, \quad (10)$$

where a and b are the parameters $(a, b > 0)$. Equation (10) is also known as Weibull distribution, see e.g. [5] and references therein. For the same data as before the fitted curves are shown in Fig. 1(d-f) providing values for the exponent a roughly between $1/3$ and $1/2$.
Thus, now the integral relating the extreme events s and damages D, Eq. (4), is over $\widetilde{p}(D)$ from Eq. (10), instead of Eq. (2)

$$\begin{aligned} P_{(s)} &= \int_{D_{(s_0)}}^{D_{(s)}} B\frac{a}{b}D^{a-1}e^{-\frac{D^a}{b}}\mathrm{d}D \\ &= 1 - Be^{-\frac{D_{(s)}^a}{b}}\,, \end{aligned} \quad (11)$$

where $B = e^{\frac{D_{(s_0)}^a}{b}}$ such that $\widetilde{p}_{(D)}$ is normalized in $D \in \left[D_{(s_0)}, D_{(s\to\infty)}\right)$. Solving for $D_{(s)}$ we find

$$D_{(s)} = \left[D_{(s_0)}^a - b\,\ln(1 - P_{(s)})\right]^{\frac{1}{a}}\,. \quad (12)$$

Finally, we insert the Gumbel distribution, i.e. Eq. (1) with $\xi = 0$, and obtain

$$D_{(s)} = \left[D_{(s_0)}^a - b\,\ln\left(1 - \exp\left[-e^{-\frac{s-\nu}{\gamma}}\right]\right)\right]^{\frac{1}{a}}\,, \quad (13)$$

which asymptotically increases as a power-law with exponent $1/a$. To make this clearer, we again employ GPD and insert Eq. (8) instead of Eq. (1) into Eq. (12). For the Gumbel case ($\xi = 0$) we find

$$D_{(s)} = \left(D_{(s_T)}^a + \frac{b}{\widetilde{\gamma}}(s - s_T)\right)^{\frac{1}{a}}\,. \quad (14)$$

Accordingly, for a between $1/3$ and $1/2$ we obtain the asymptotic power-law relation $D_{(s)} \sim s^3$ or $D_{(s)} \sim s^2$. Thus, if we assume stretched exponential distributed damages as well as extreme events following the Gumbel distribution, then the damages caused by such natural disasters must (asymptotically) depend as a power-law on the magnitude of the corresponding extreme events.

Discussion

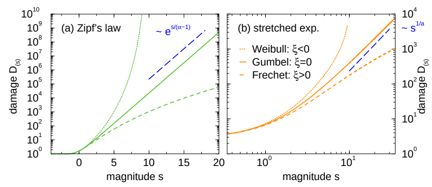

Figure 3

In summary, we characterize distributions of recorded flood damages, argue that they are caused by extreme events, and employ density transformation to deduce damage functions relating both. For Gumbel distributed extreme events ($\xi = 0$) we find

$$D_{(s)} \sim \begin{cases} e^{\frac{s}{\alpha-1}} & \text{for } \widetilde{p}_{(D)} \sim D^{-\alpha} \text{ with } \alpha > 1 \\ s^{\frac{1}{a}} & \text{for } \widetilde{p}_{(D)} \sim \frac{a}{b}D^{a-1}e^{-\frac{D^a}{b}} \text{ with } a > 0\,. \end{cases} \quad (15)$$

The functional forms are illustrated in Fig. 3, which also includes the cases $\xi \neq 0$. For power-law distributed damages and Weibull distributed extreme events ($\xi < 0$) in average the damage increases faster than exponentially with the magnitude. For Fréchet distributed extreme events ($\xi > 0$) the opposite is the case. Intuitively, since in the Weibull case the extreme events have an a bounded tail, a steeper damage function is needed to result in the same damage distribution as the Gumbel case [Fig. 3(a)]. In the Fréchet case, which has a fatter tail than the Gumbel distribution, a less steep damage function is sufficient to result in the same damage distribution as the Gumbel case. Similar arguments hold for stretched exponential damages [Fig. 3(b)].
The obtained results can have important applications and implications for the assessment of damages due to natural disasters. In particular, in the scope of climate change, knowledge about upcoming costs is demanded [6]. Changing climate comes along with evolving extreme value statistics and corresponding damage functions need to be adapted accordingly.

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