



POTSDAM INSTITUTE FOR  
CLIMATE IMPACT RESEARCH



# On the estimation of damages due to coastal floods

**Diego Rybski**

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Olivia Roithmeier; Carsten Walther;  
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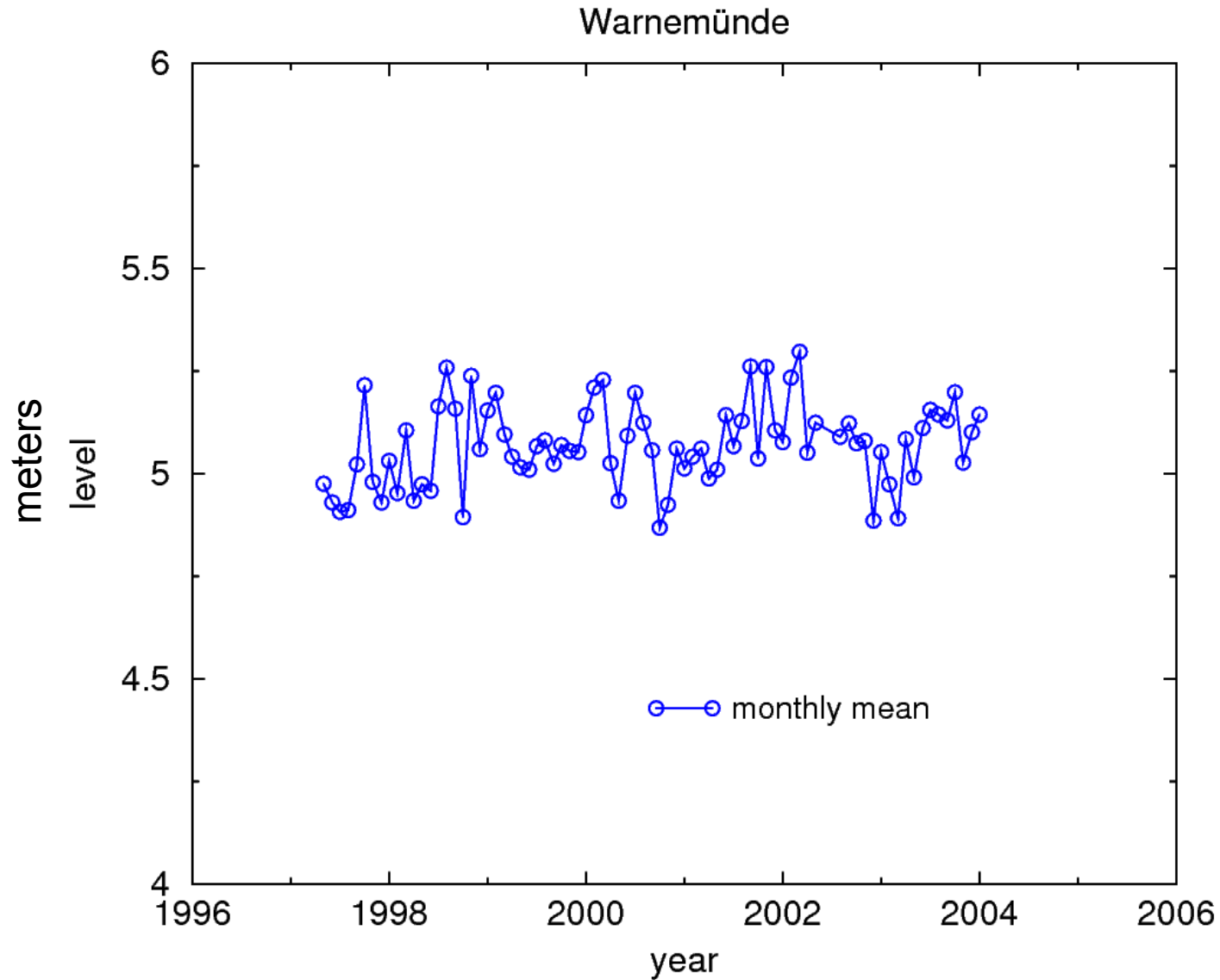
8.12.2011 – 14:00-14:20  
Room 304  
NG43C-02

**NG43C:** Scaling Functions, Trends, Correlations,  
and Cross-Correlations in Geosciences and  
Their Use in Forecasting Natural Hazards I

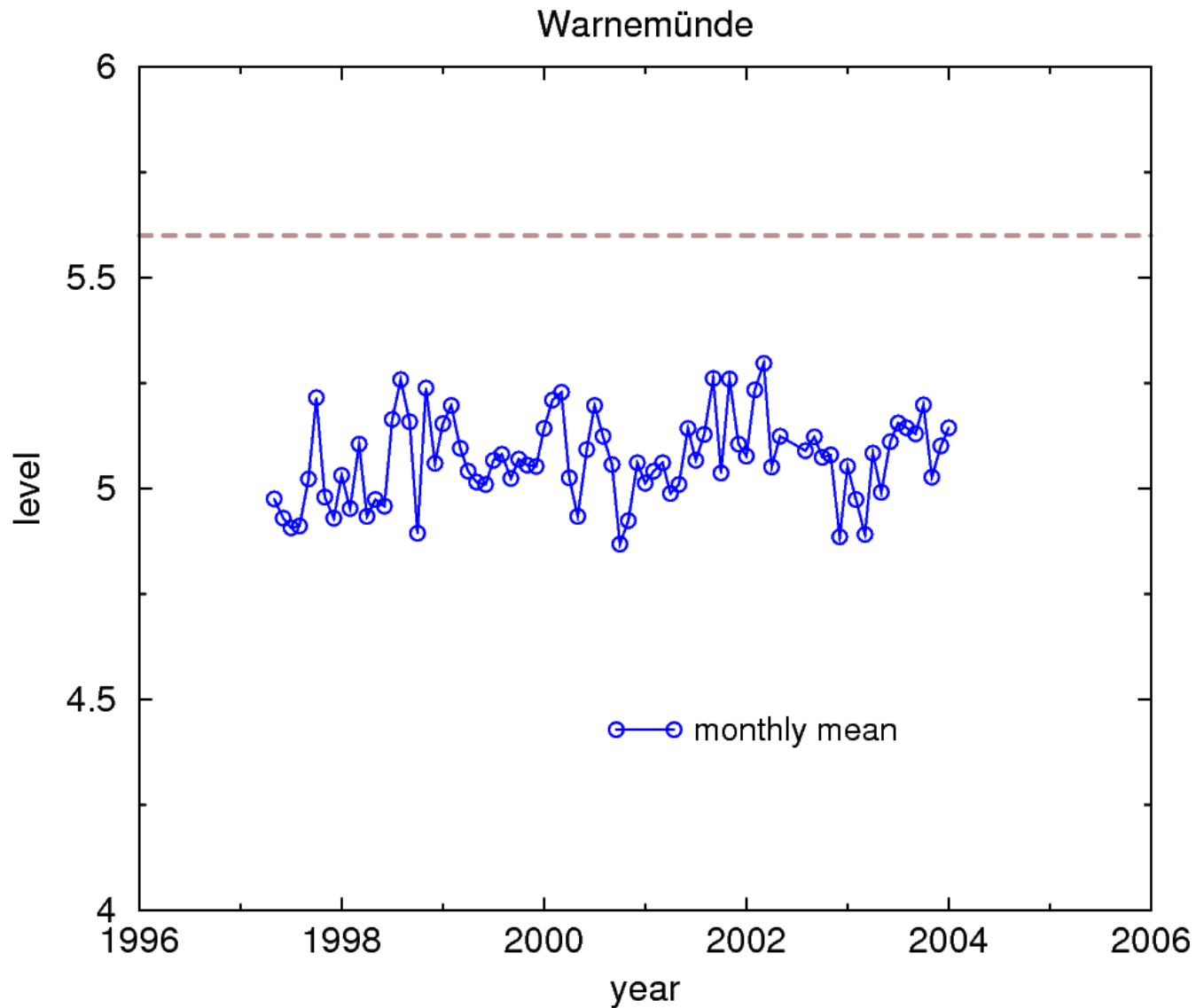
# Outline

1. Extremes matter
2. Damage functions
3. Expected damages and uncertainty

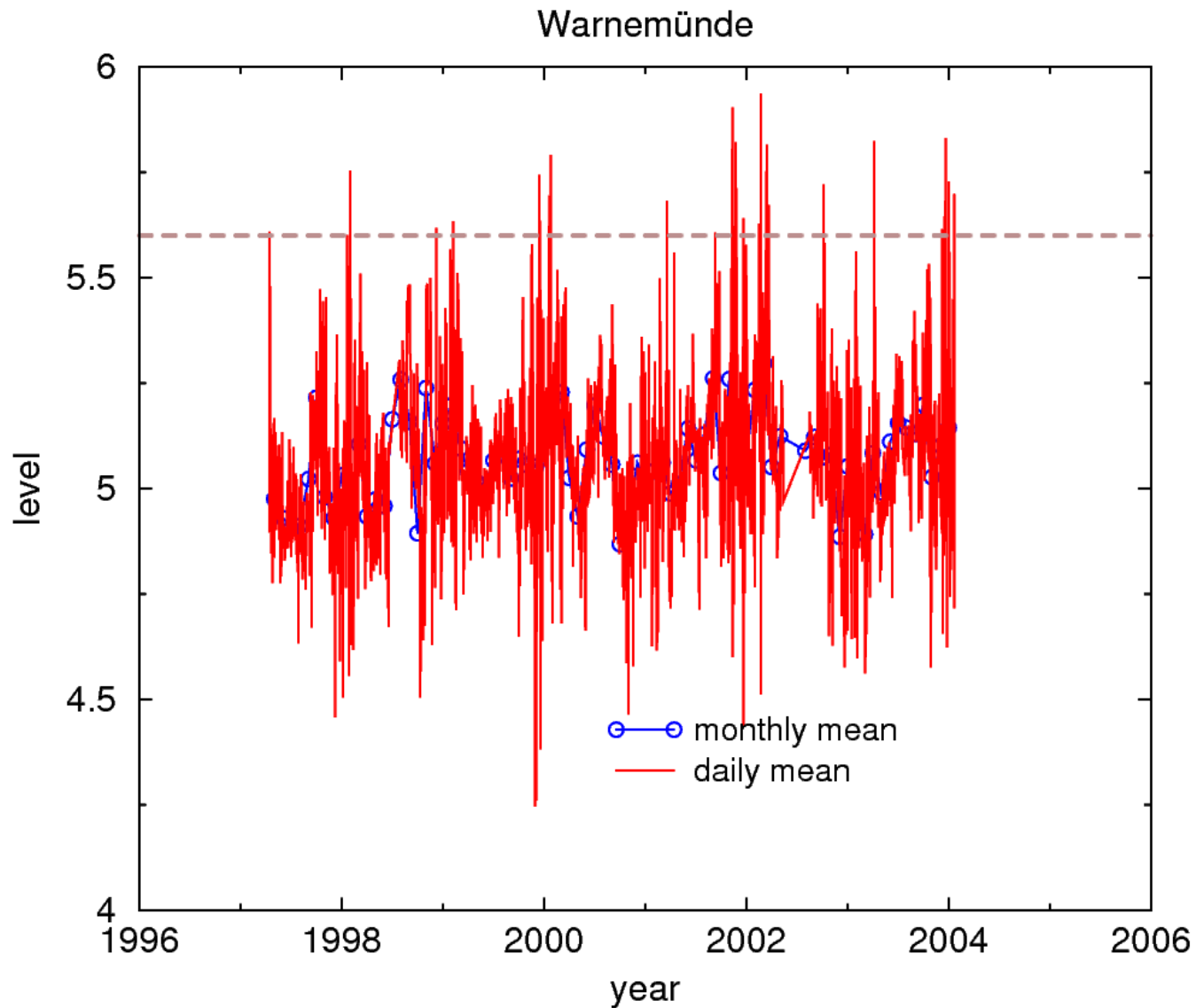
# Sea level rise and fluctuations



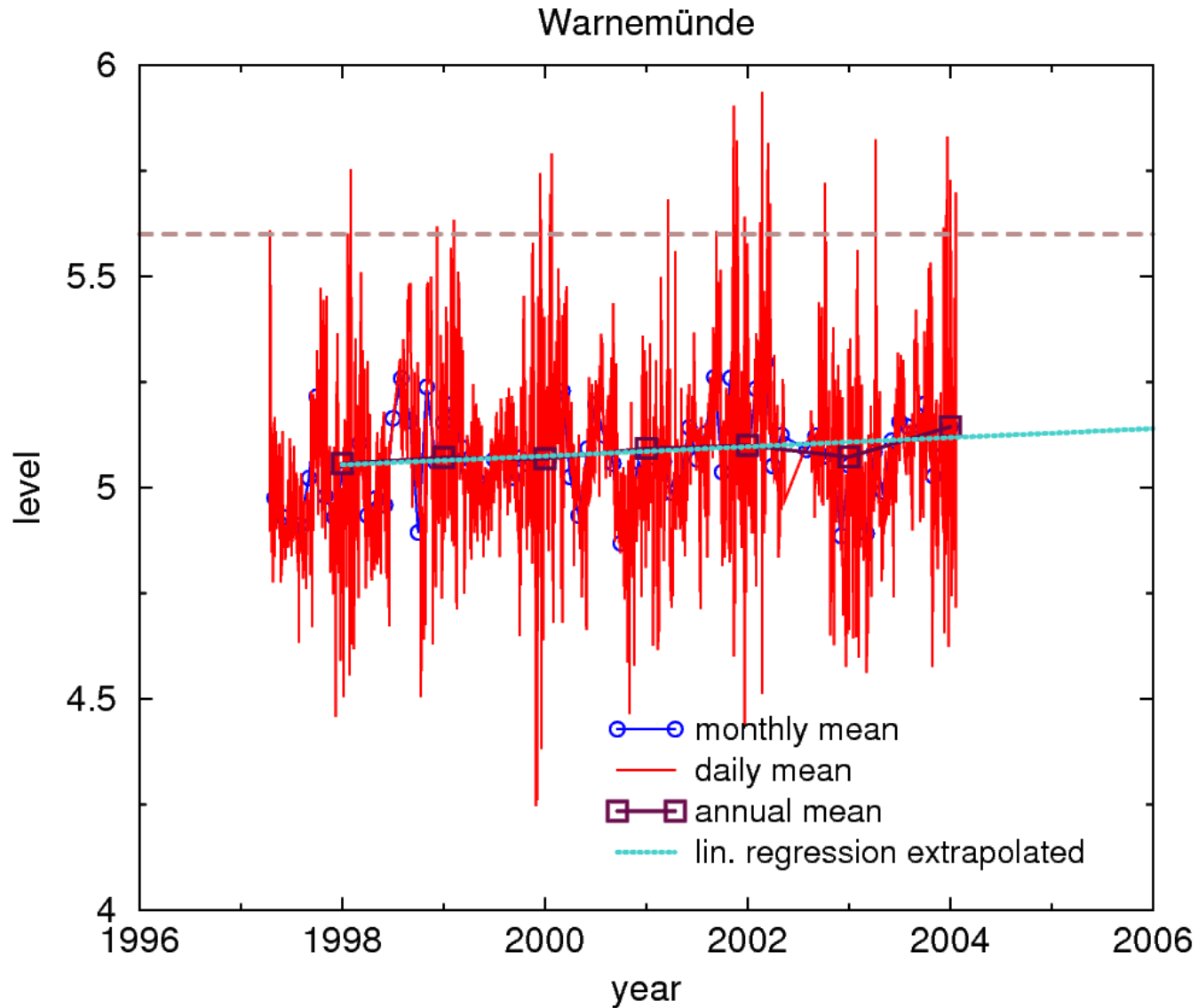
# Sea level rise and fluctuations

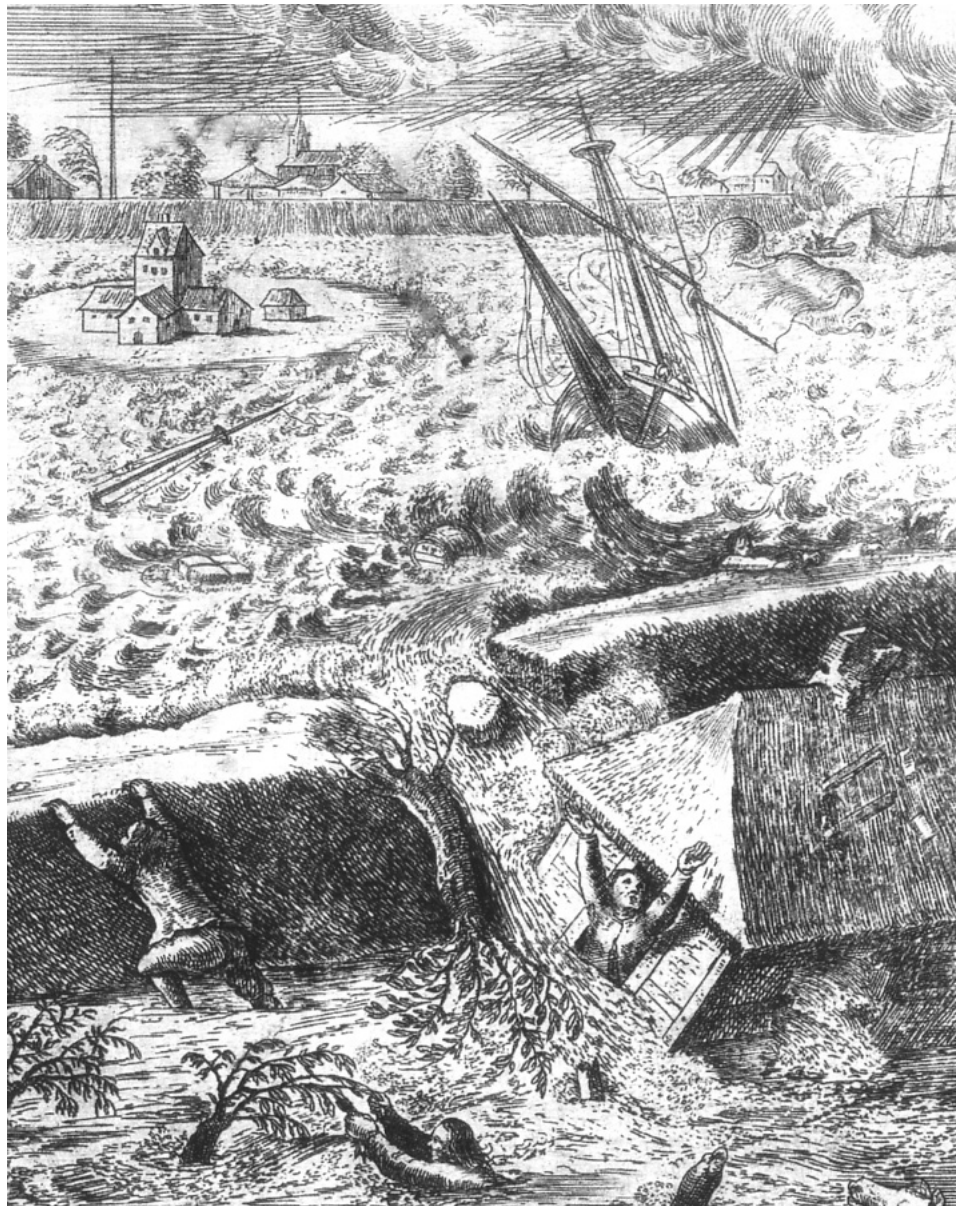


# Sea level rise and fluctuations



# Sea level rise and fluctuations





wikipedia: Kupferstich "Deichbruch" von Winterstein 1661

Salacgriva (Latvia)



# Motivation

How to estimate damages from (coastal) floods?

How do they change with sea-level-rise?

How are they influenced by protection measures?

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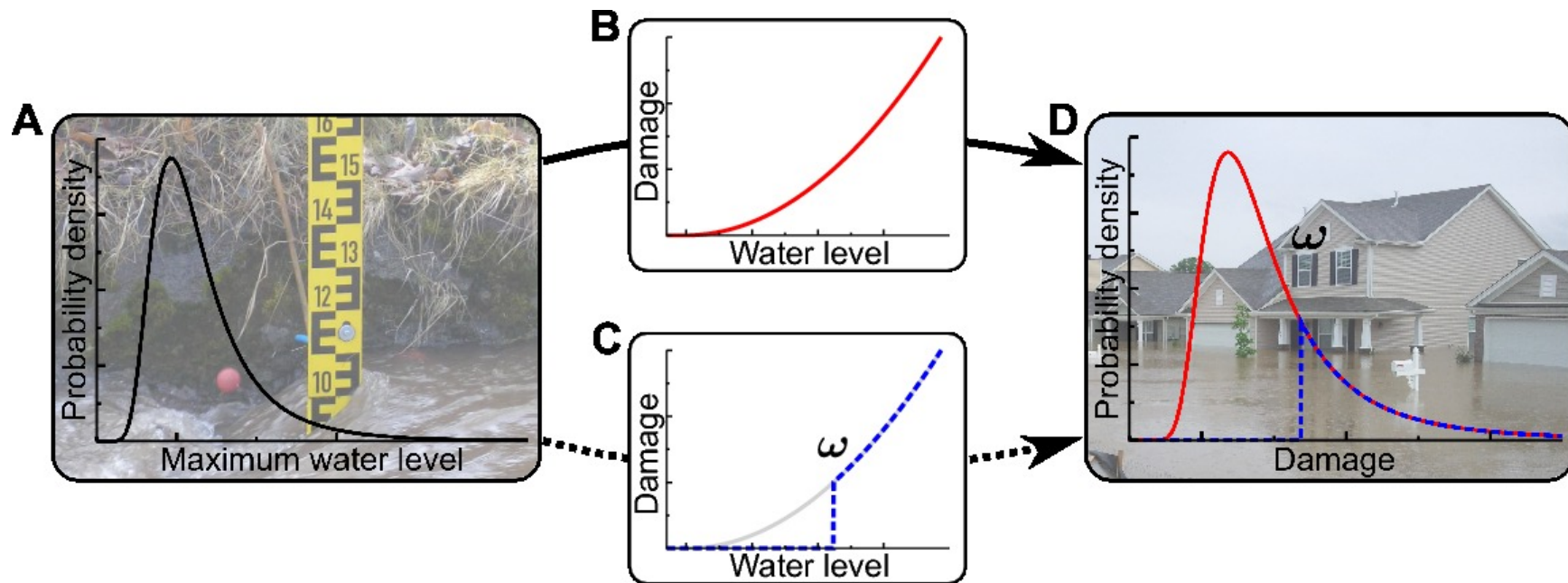
- consider distribution of extremes
- combine with damage function
- study distribution of damages
- dependence on GEV-parameters and protection

# Motivation

How to estimate damages from (coastal) floods?

How do they change with sea-level-rise?

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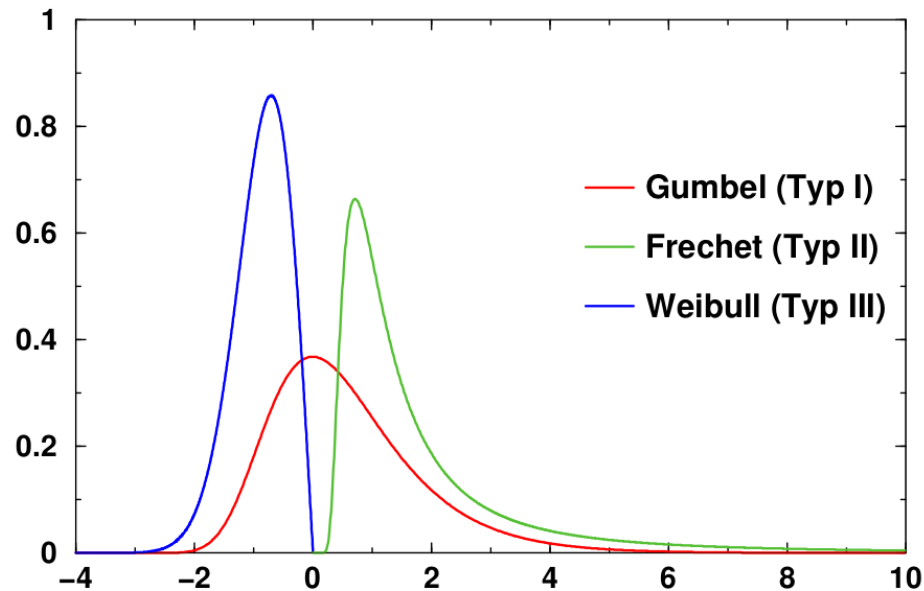


# Generalized Extreme Value distributions

The GEV distributions, expressing the probability that the maximum of a sample is beneath the value  $s$ , are given by:

$$P_{(s)}^{\text{GEV}} = \begin{cases} \exp \left[ - \left( 1 + \xi \frac{s - \nu}{\gamma} \right)^{-\frac{1}{\xi}} \right] & \text{for } \xi \neq 0 \\ \exp \left[ -e^{-\frac{s - \nu}{\gamma}} \right] & \text{for } \xi = 0. \end{cases} \quad (1)$$

They are defined on  $\left\{ s : 1 + \xi \frac{s - \nu}{\gamma} > 0 \right\}$  and have a location parameter,  $\nu \in \mathbb{R}$ , a scale parameter,  $\gamma \in \mathbb{R}^+$ , as well as a shape parameter,  $\xi \in \mathbb{R}$ . According to the shape, one distinguishes three cases: (i) the Gumbel distribution ( $\xi = 0$ ), (ii) the heavy-tailed Fréchet distribution ( $\xi > 0$ ), and (iii) the bounded-tailed *reversed* Weibull distribution ( $\xi < 0$ ).



# Damage functions

intuitively: the higher the flood, the more damage

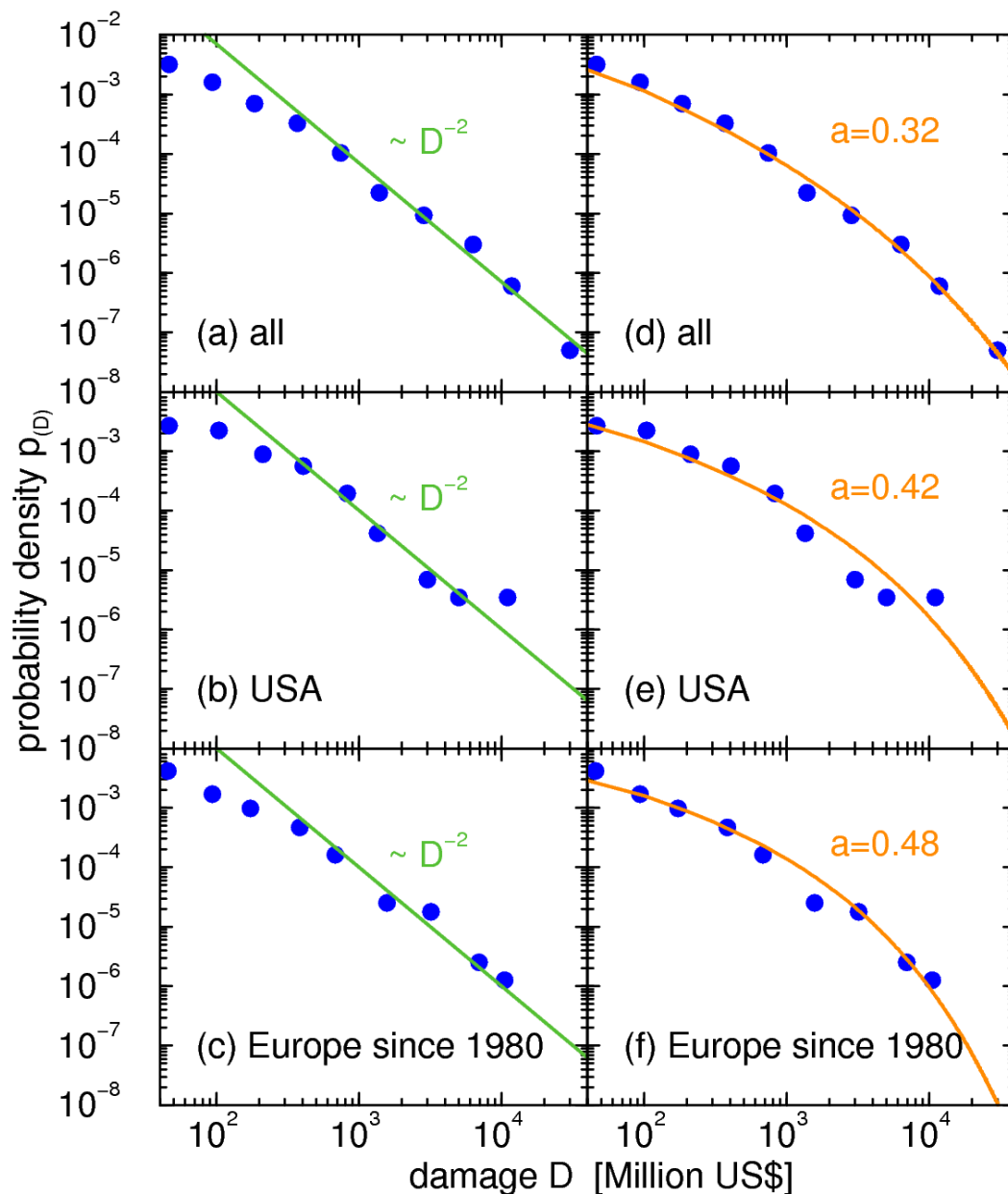
damage function: typical damage for flood of certain height

problem: how to determine damage functions?

- empirical data (here: indirectly)
- case study

later: assume power-law

# Damage functions from damage records



Zipf's law

stretched exponential

Which damage function is required so that GEV transforms into observed distribution of damages?

→ density transformation


data: CRED [EM-DAT, 2009], damages due to floods worldwide in the years 1950-2008

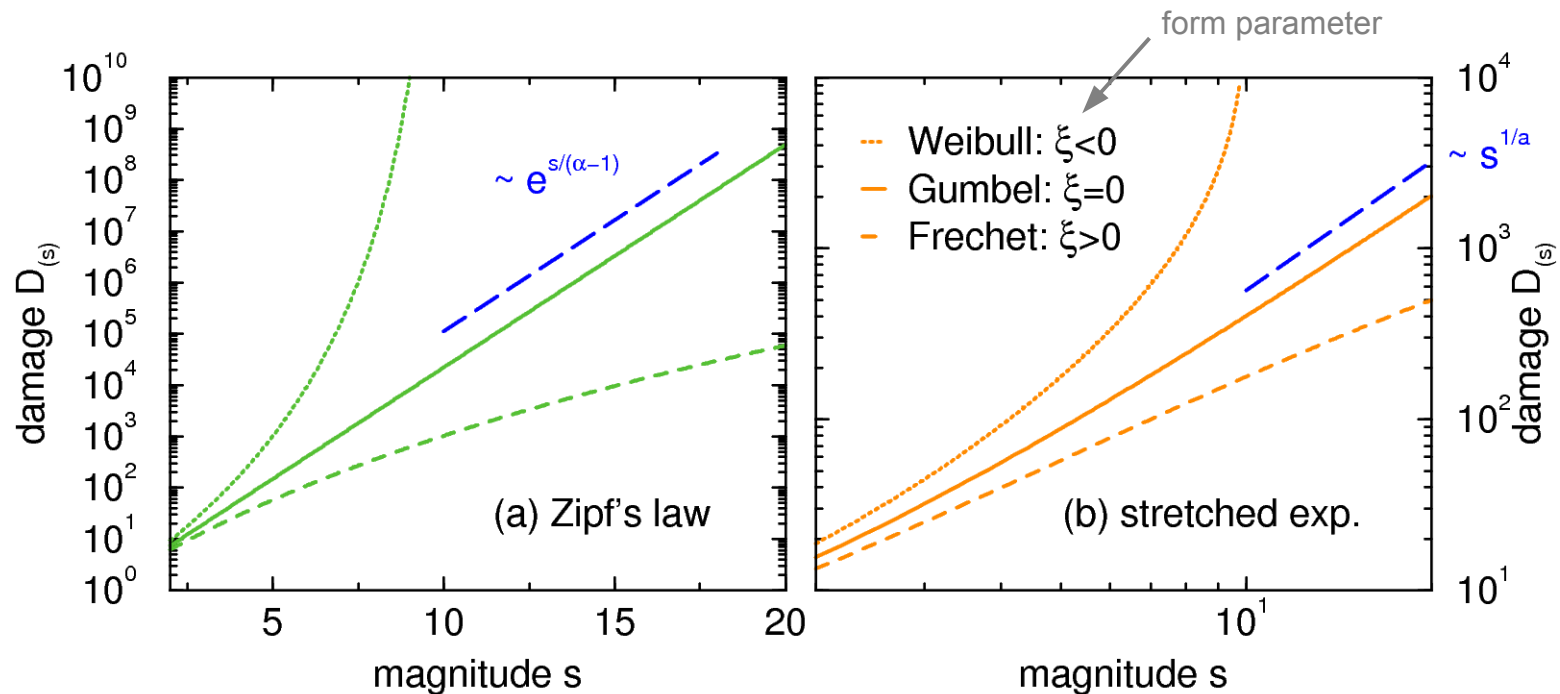
# Damage functions from damage records

Gumbel: 
$$D_{(s)} \sim \begin{cases} e^{\frac{s}{\tilde{\gamma}(\alpha-1)}} & \text{for } \tilde{p}_{(D)} \sim D^{-\alpha} \text{ with } \alpha > 1 \\ \left(\frac{1}{\tilde{\gamma}}s\right)^{\frac{1}{a}} & \text{for } \tilde{p}_{(D)} \sim \frac{a}{b}D^{a-1}e^{-\frac{D^a}{b}} \text{ with } a > 0 \end{cases}$$

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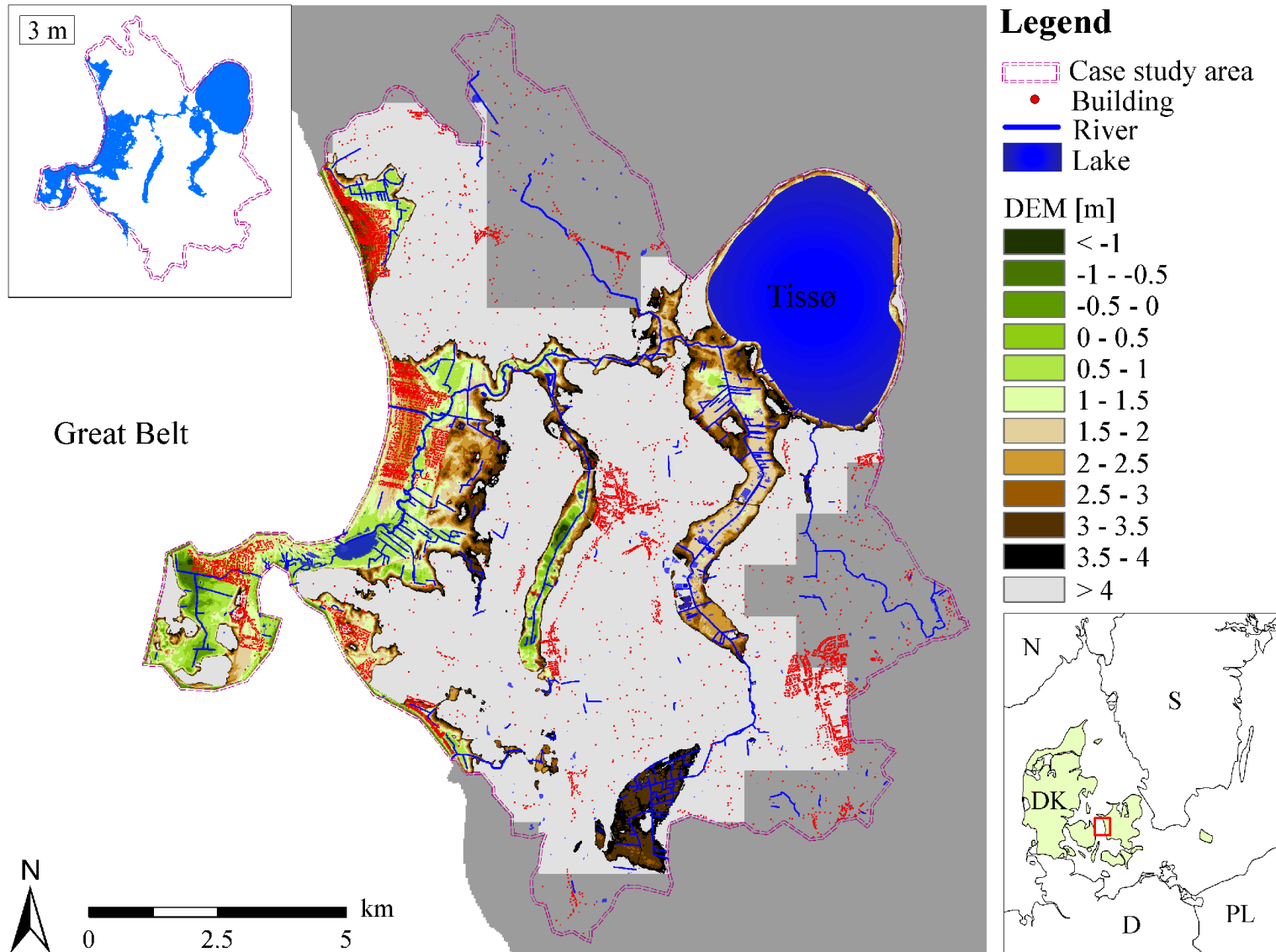

 scale parameter



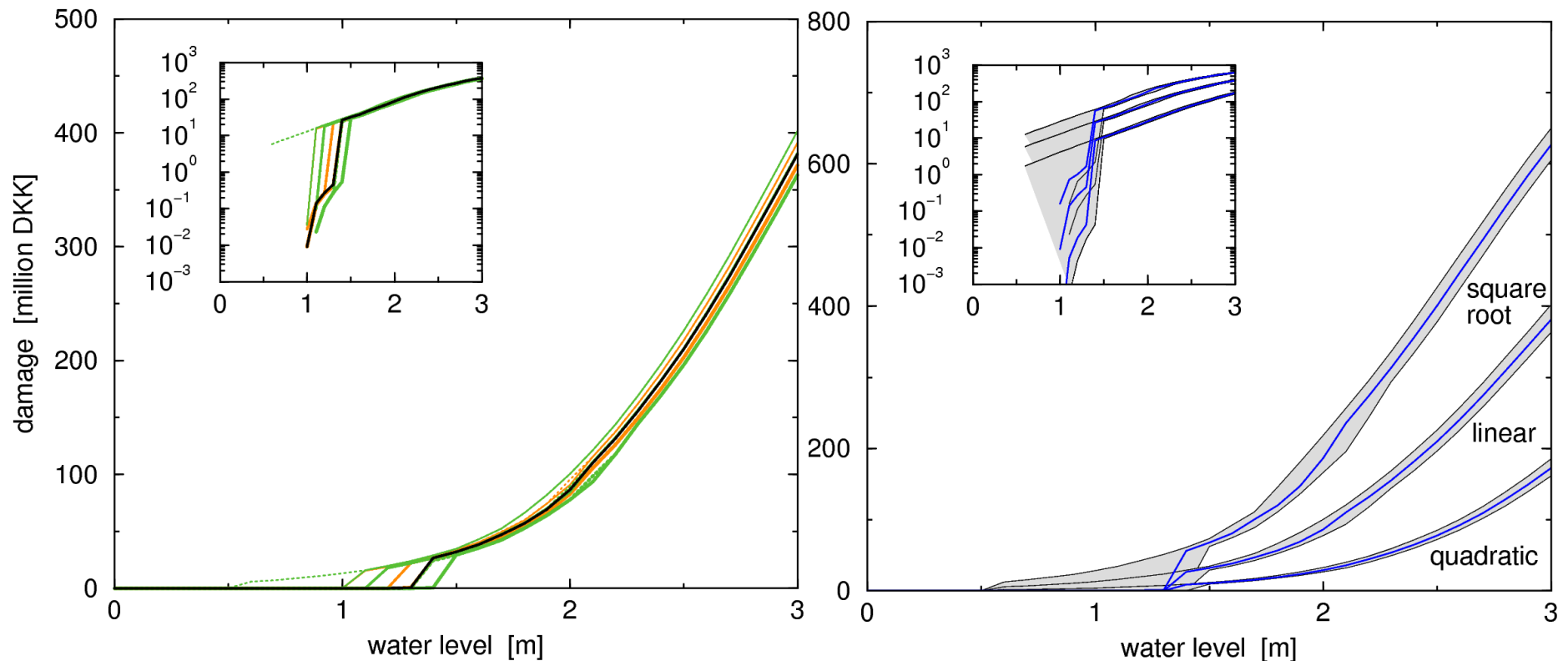


# Damage functions from case study

## - Kalundborg (DK)

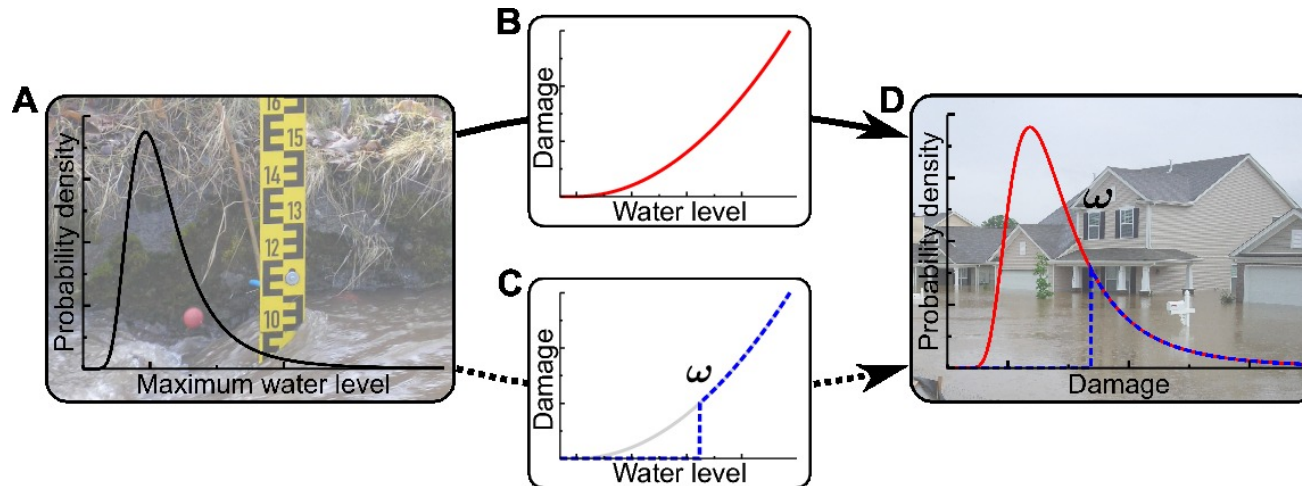


# Damage functions from case study - Kalundborg (DK)



hydro-dynamical modeling  
more convenient: flood fill

# Expected damages and uncertainty



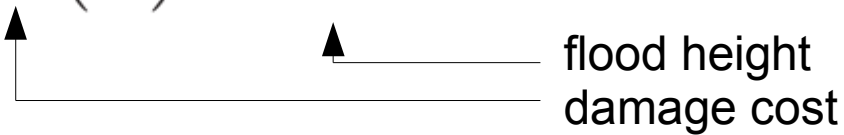
damage costs	location $\mu$	scale $\sigma$	protection height $\omega$
E expectation value			
STD standard deviation			



e.g. changing weather patterns  
e.g. sea-level-rise


# Expected damages and uncertainty - as a function of the *location*

damage function:

$$D(x) \sim x^\gamma$$


flood height  
damage cost

expectation value:

$$E(C) \sim \mu^\gamma$$


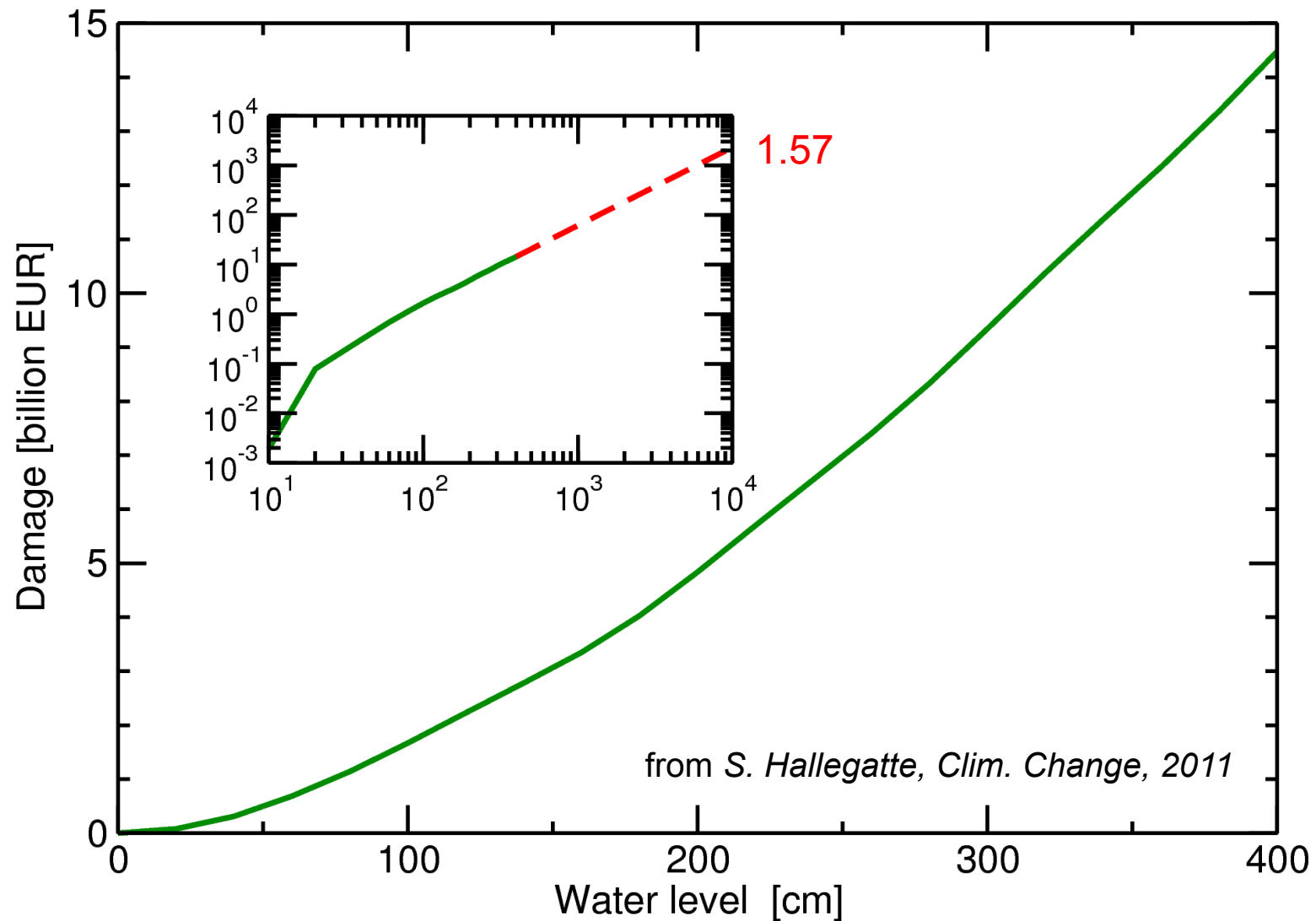
location

standard deviation:

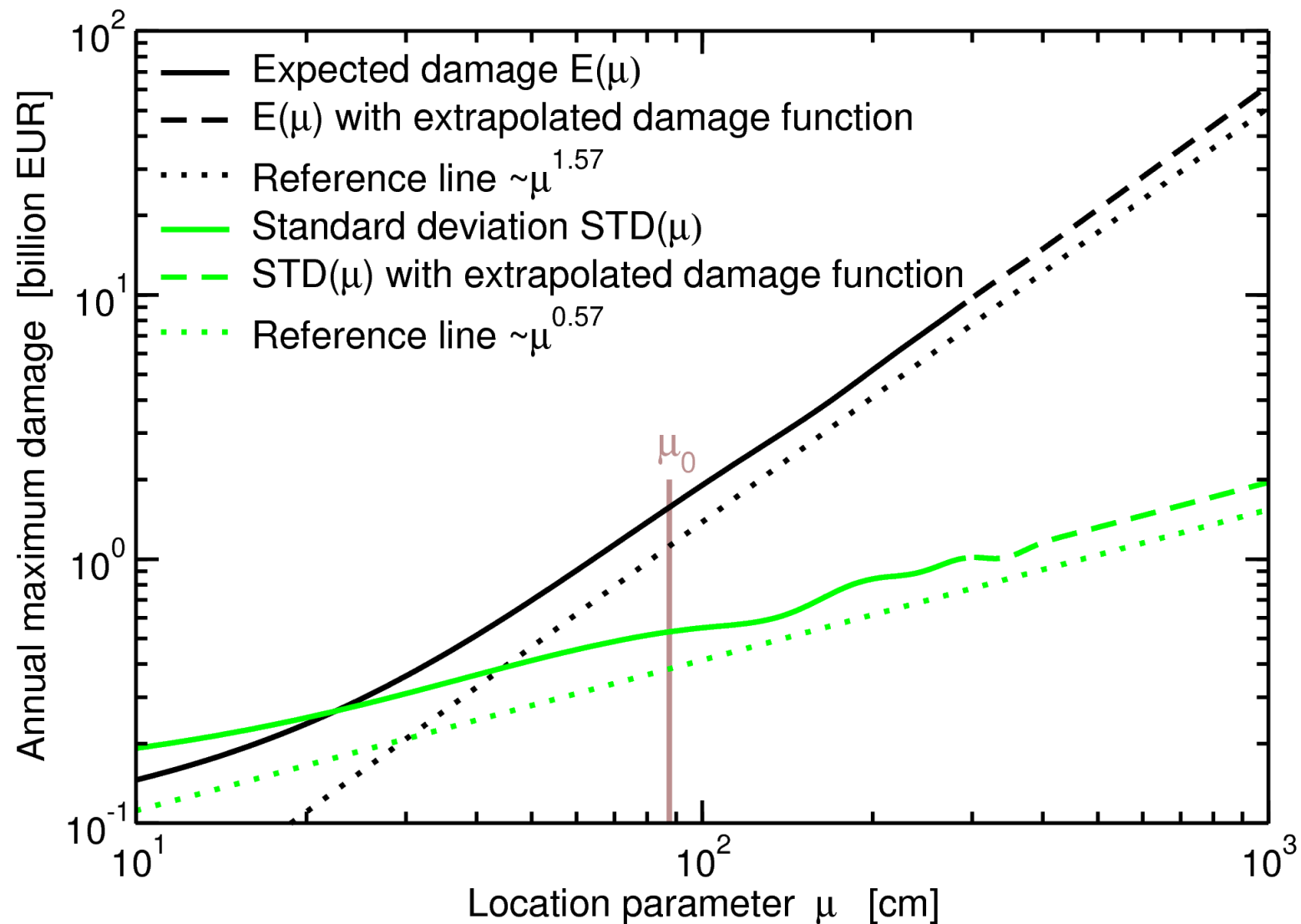
$$\text{STD}(C) \sim \mu^{\gamma-1}$$

asymptotically  
independent from GEV-type (!)  
relative uncertainty *decreases*  
damage function exponent is decisive

# Expected damages and uncertainty - case study Copenhagen

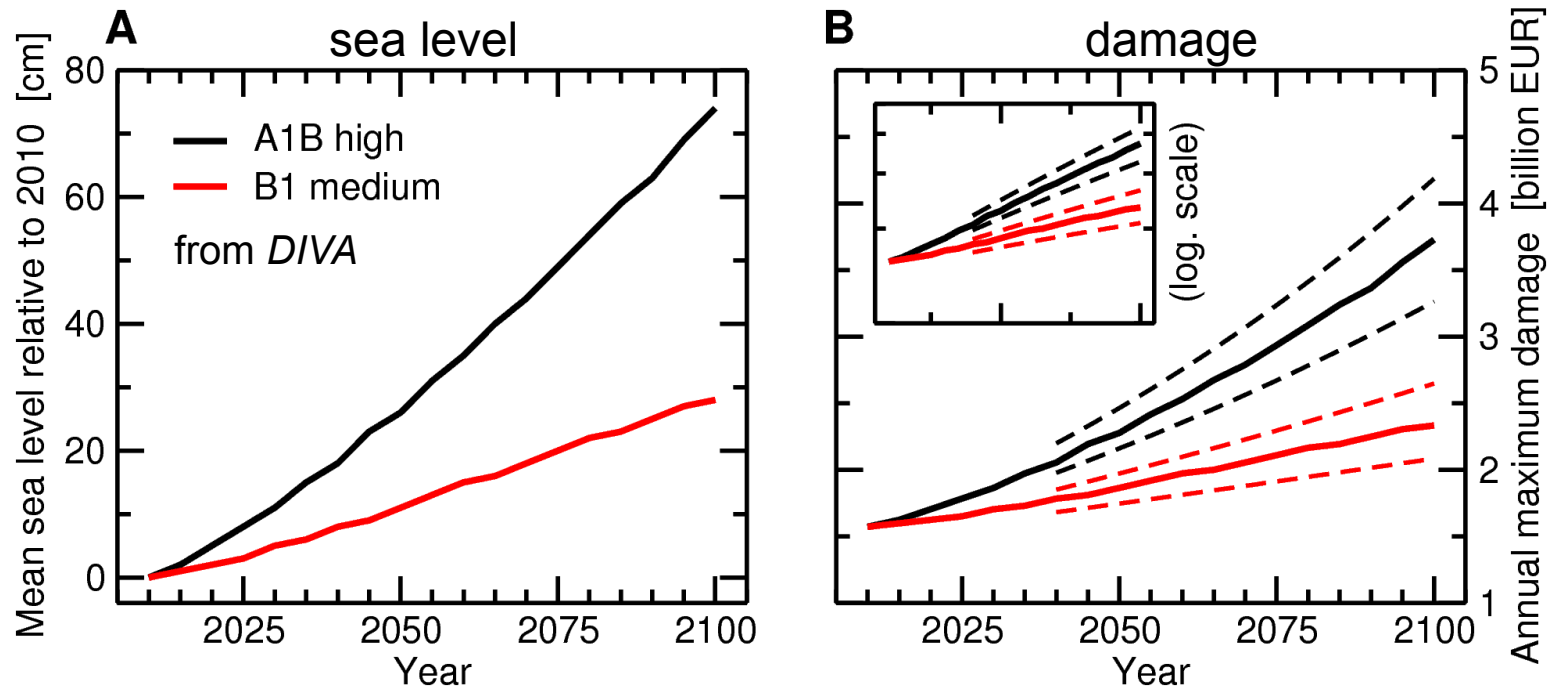


# Expected damages and uncertainty - case study Copenhagen



# Expected damages and uncertainty

## - case study Copenhagen



using local sea level projections from *DIVA*-tool

temporal evolution of expected damage

# Expected damages and uncertainty - as a function of the *scale*

expectation value:  $E(C) \sim \sigma^\gamma$

▲———— scale

standard deviation:  $STD(C) \sim \sigma^\gamma$

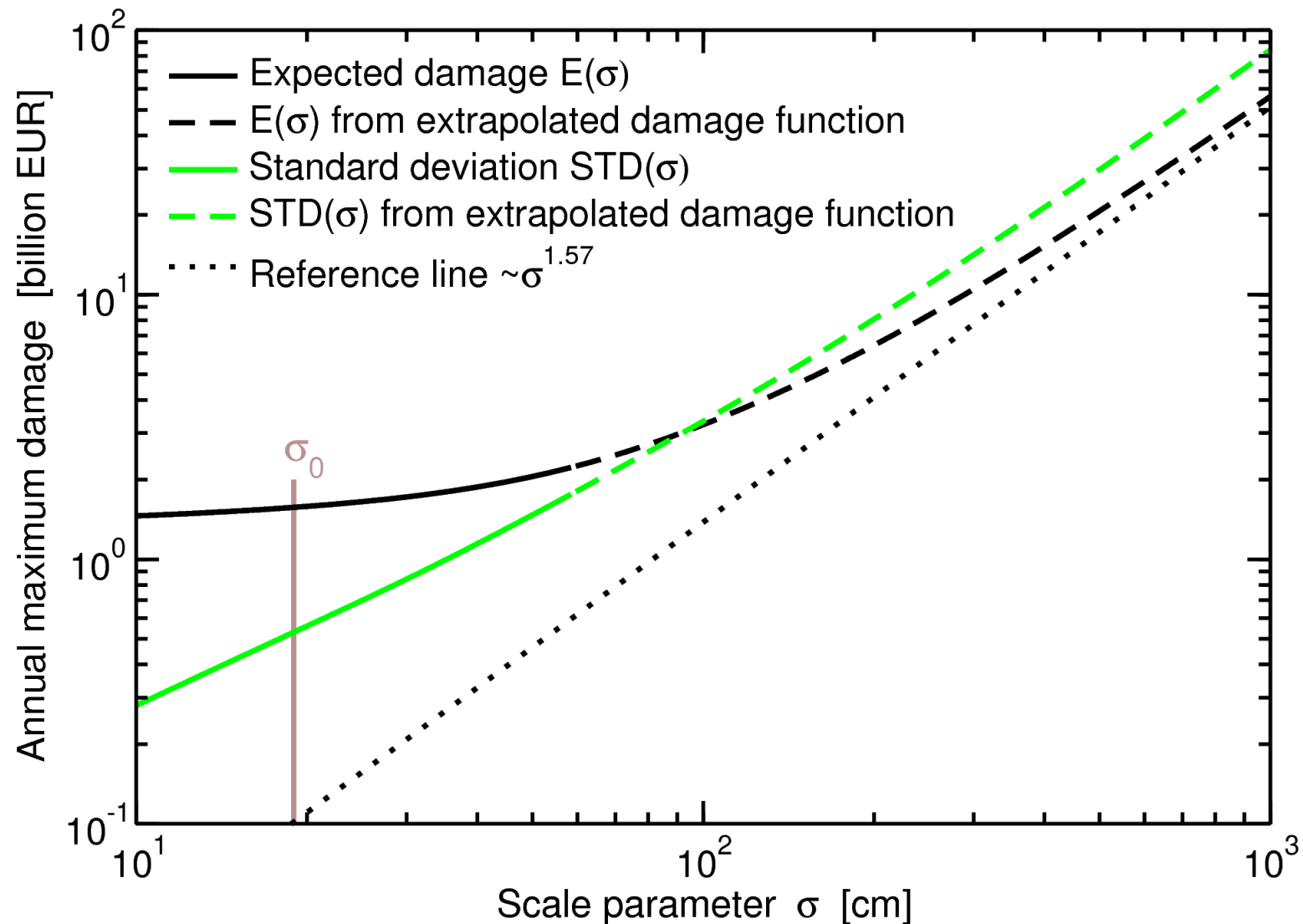
asymptotically  
independent from GEV-type (!)  
relative uncertainty is *constant*  
damage function exponent is decisive

$$STD(C) \sim \mu^{\gamma-1}$$



# Expected damages and uncertainty

## - case study Copenhagen



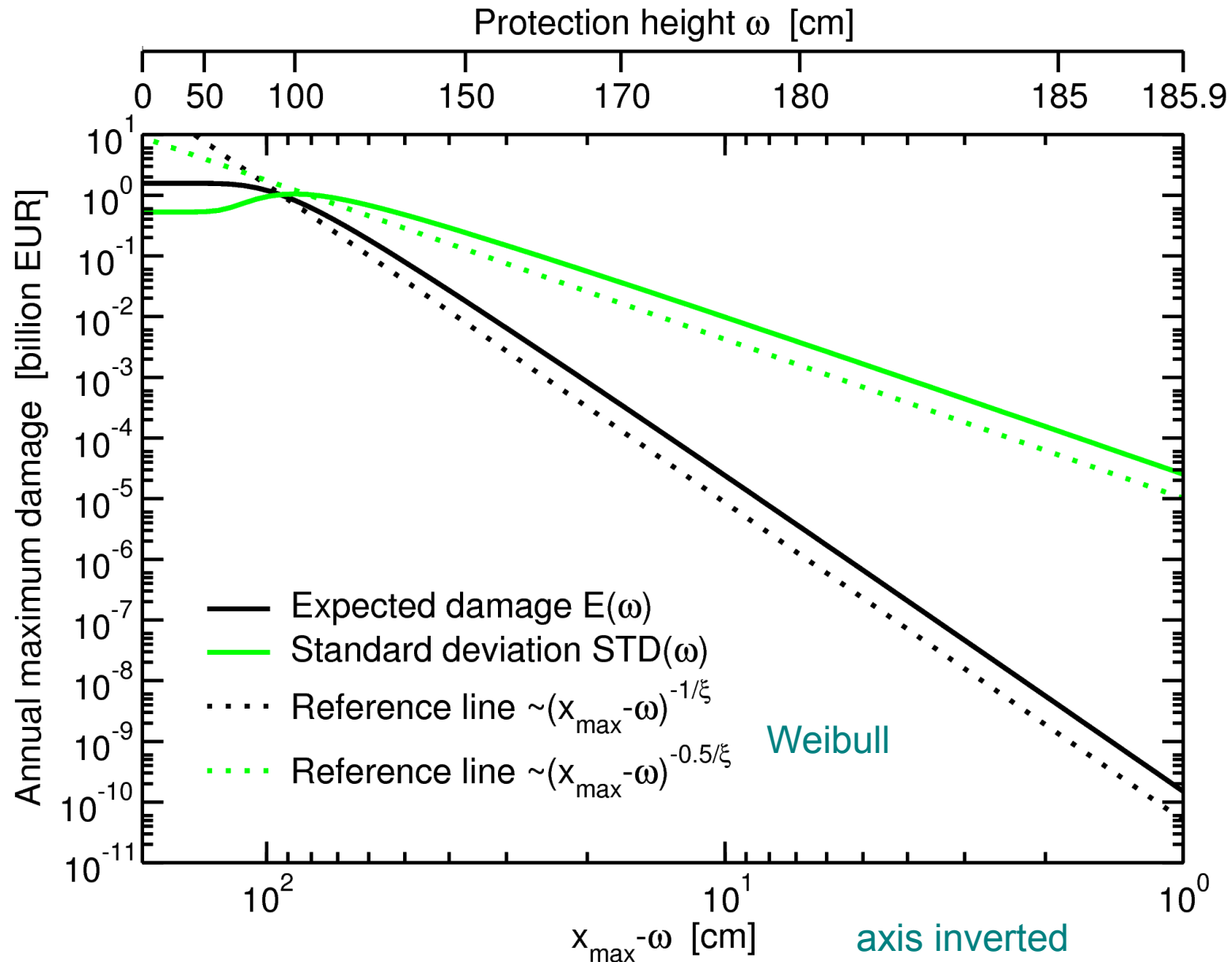
# Expected damages and uncertainty - as a function of *protection height*

I.e. how does the expected annual damage decrease with increasing protection height?

Gumbel:	$E(C) \sim \omega^\gamma e^{-\omega/\sigma}$	scale parameter
Frechet:	$E(C) \sim \omega^{\gamma-1/\xi}$	form parameter
Weibull:	$E(C) \sim (x_{\max} - \omega)^{-1/\xi}$	independent of damage function exponent

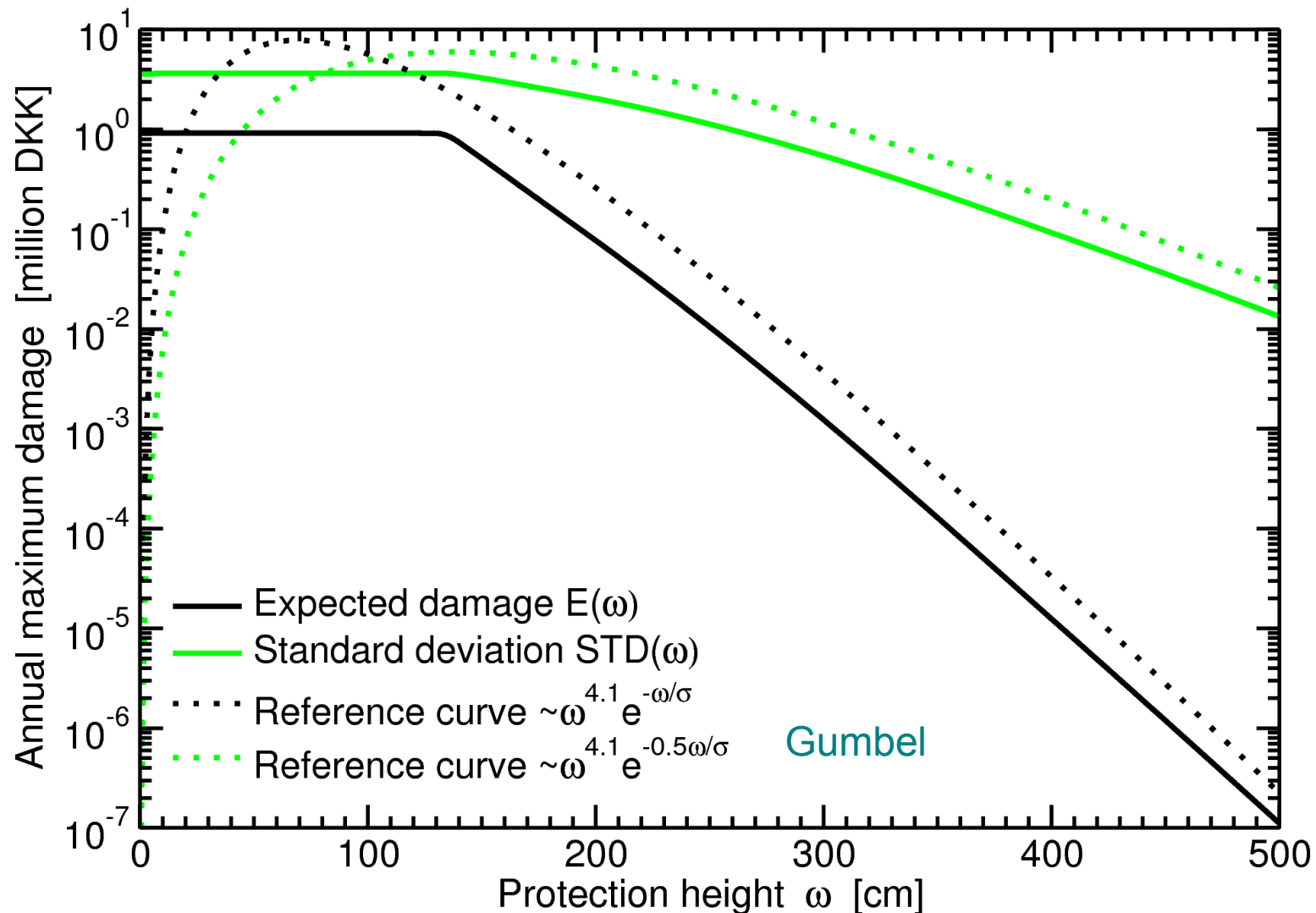
asymptotically  
3 fundamentally different cases  
relative uncertainty is *increases*

# Expected damages and uncertainty - case study Copenhagen



# Expected damages and uncertainty

## - case study *Kalundborg*



# Expected damages and uncertainty

## - overview

	location $\mu$	scale $\sigma$	protection height $\omega$
E	$\sim \mu^\gamma$	$\sim \sigma^\gamma$	$\sim \omega^\gamma e^{-\omega/\sigma}$ if $\xi = 0$ $\sim \omega^{\gamma-1/\xi}$ if $\xi > 0$ $\sim (x_{\max} - \omega)^{-1/\xi}$ if $\xi < 0$
STD	$\sim \mu^{\gamma-1}$	$\sim \sigma^\gamma$	$\sim \omega^\gamma e^{-0.5\omega/\sigma}$ if $\xi = 0$ $\sim \omega^{\gamma-0.5/\xi}$ if $\xi > 0$ $\sim (x_{\max} - \omega)^{-0.5/\xi}$ if $\xi < 0$

differ only by the factor 0.5  
in the exponent

uncertainty only due to the fact  
that one does not know when  
the extremes take place  
(lower estimate)

Vorstellung einer Bracke oder Durch-Bruch eines Damms dadurch daß Landt von jinnen über -  
Schwemmet wirdt.



Vorstellung einer Bracke oder Durch-Bruch eines Dammes dadurch dass Landt vor jinnen überschwemmet wirdt, wikipedia, prb. 1718

# Acknowledgments

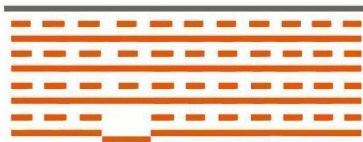
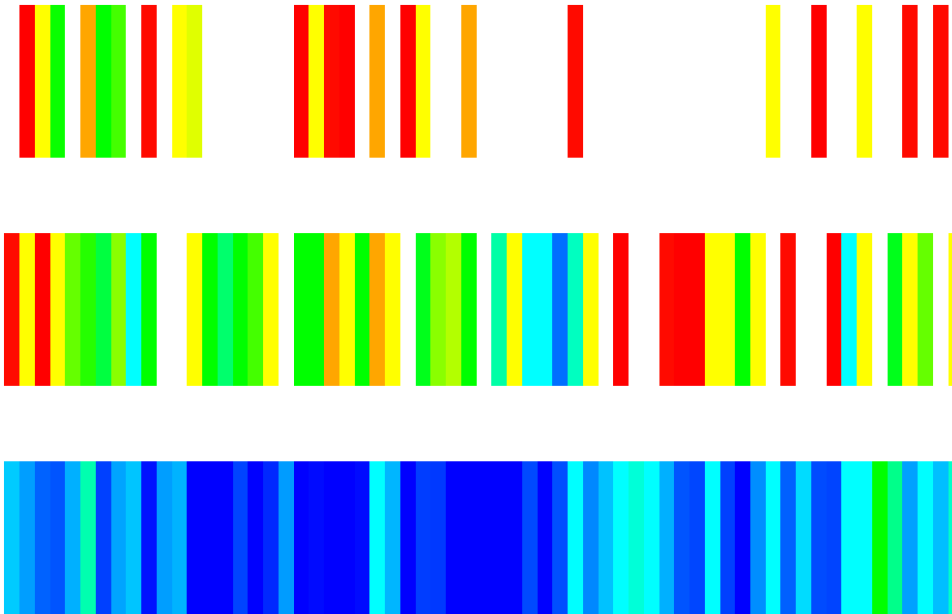


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# Thank you for your attention.



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