Benjamin Levich Institute

About human activity, long-term memory, and Gibrat's law

Diego Rybski,
Sergey V. Buldyrev, Shlomo Havlin,
Fredrik Liljeros, Hernán A. Makse

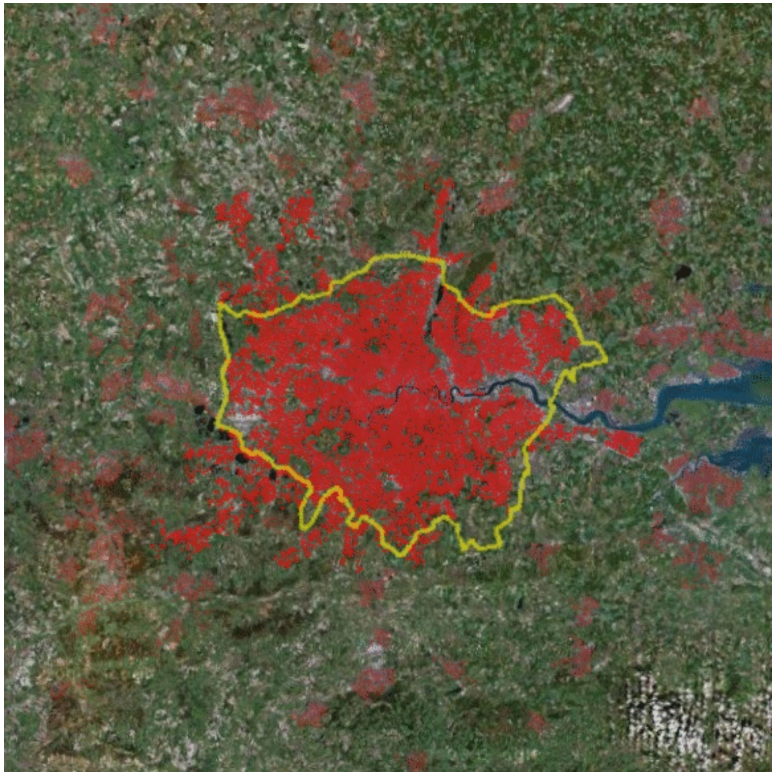
AGSOE 17.2
DPG Frühjahrstagung 2009
Dresden; BAR 205
26.3.2009; 10:45 – 11:15

the City College of New York
OF THE CITY UNIVERSITY OF NEW YORK

Motivation

City Growth, see:

- Rozenfeld HD, et al., PNAS 105, 2008
- “New Laws of City Growth” AGSOE 8.1 24.3.2009



Pioneering work

Scaling behaviour in the growth of companies

Michael H. R. Stanley*, Luís A. N. Amaral*, Sergey V. Buldyrev*, Shlomo Havlin*†, Heiko Leschhorn*, Philipp Maass*, Michael A. Salinger‡ & H. Eugene Stanley*

* Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 02215, USA
† Minerva Center and Department of Physics, Bar-Ilan University, Raman Gan, Israel
‡ Department of Finance and Economics, School of Management, Boston University, Boston, Massachusetts 02215, USA

A SUCCESSFUL theory of corporate growth should include both the external and internal factors that affect the growth of a company[1–18]. Whereas traditional models emphasize production-related influences such as investment in physical capital and in research and development[18], recent models[10–20] recognize the equal importance of organizational infrastructure. Unfortunately, no exhaustive empirical account of the growth of companies exists by which these models can be tested. Here we present a broad, phenomenological picture of the dependence of growth on company size, derived from data for all publicly traded US manufacturing companies between 1975 and 1991. We find that, for firms with similar sales, the distribution of annual (logarithmic) growth rates has an exponential form; the spread in the distribution of rates decreases with increasing sales as a power law over seven orders of magnitude. A model wherein the probability of a company's growth depends on its past as well as present sales accounts for the former observation. As the latter observation applies to companies that manufacture products of all kinds, organizational structures common to all firms might well be stronger determinants of growth than production-related

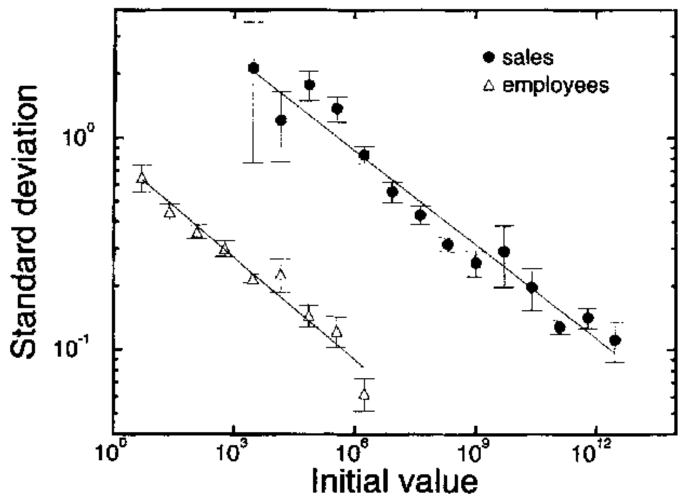
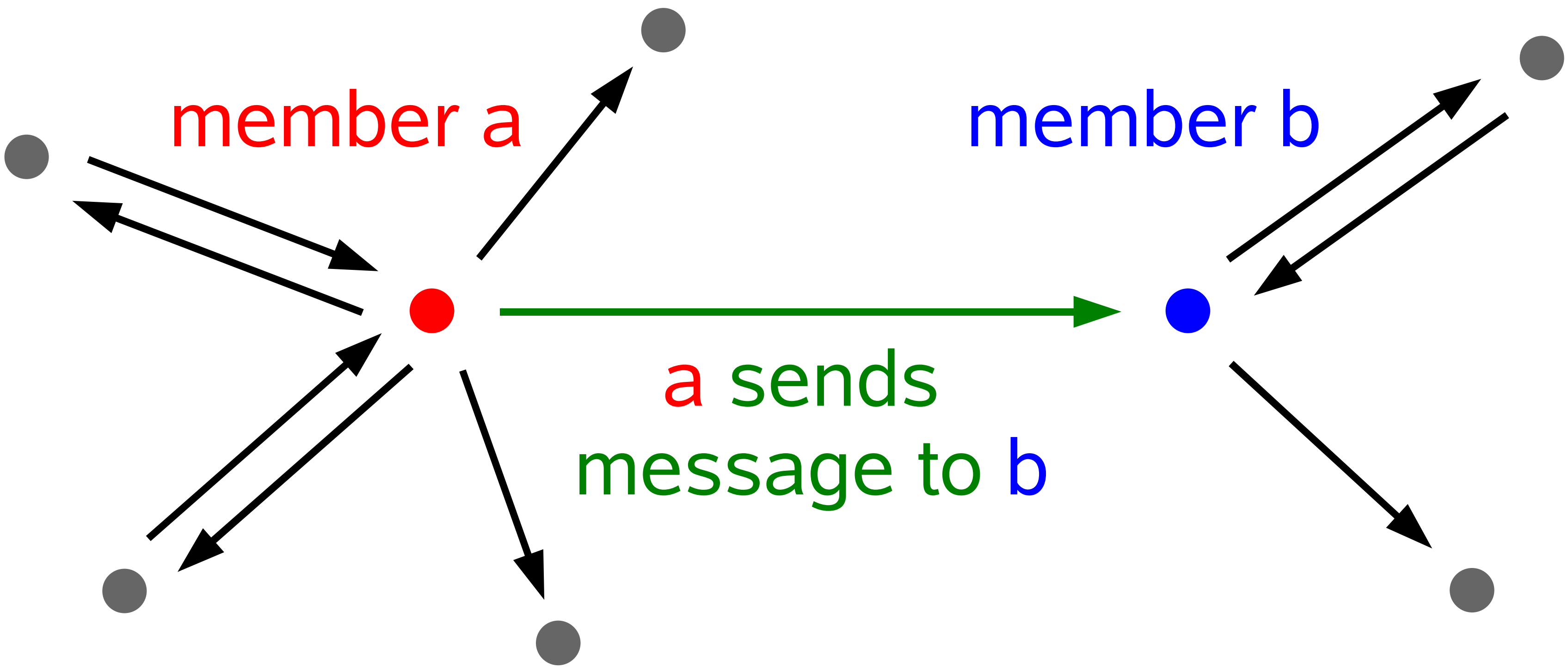


FIG. 2 Standard deviation of the one-year growth rates of the sales (circles) and of the one-year growth rates of the number of employees (triangles) as a function of the initial values. The solid lines are least-square fits to the data with slopes $\beta = 0.15 \pm 0.03$ for the sales and $\beta = 0.16 \pm 0.03$ for the number of employees. We also show error bars of one standard deviation about each data point. These error bars appear asymmetric as the ordinate is a log scale.

NATURE · VOL 379 · 29 FEBRUARY 1996

Motivation

online community: members sending messages



either following an existing link $m_{\mathrm{a}} \longrightarrow m_{\mathrm{a}} + 1$
or creating a new one $k_{\mathrm{a}}^{\mathrm{out}} \longrightarrow k_{\mathrm{a}}^{\mathrm{out}} + 1$

=> **growth process**

Outline

0. motivation
1. online community data
2. growth process
3. temporal correlations
4. missing link
5. conclusions

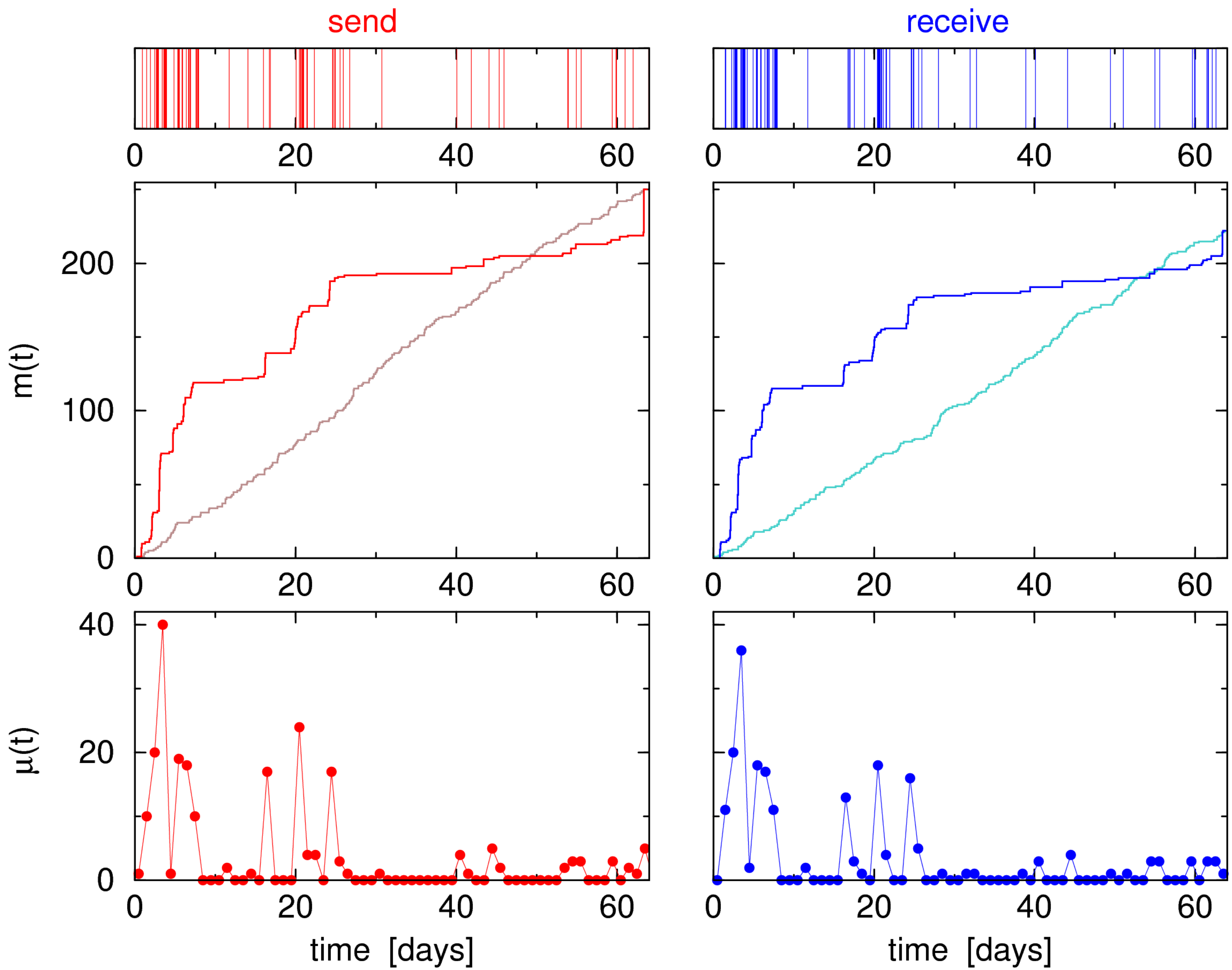
Online community data

online community 1 (OC1):
- 80,000 members
- 12.5 million messages
- 63 days

online community 2 (OC2):
- 30,000 members
- 500,000 messages
- 492 days

both are dating-communities
also used for social interaction in general
completely anonymous

Typical activity (OC1)



Growth process

for each member:

cumulative number of messages $m(t)$

logarithmic growth rate $r = \ln \frac{m_1}{m_0}$

between two time-steps t_0, t_1

two quantities:

conditional average growth $\langle r(m_0) \rangle = \langle r | m_0 \rangle$

cond. standard deviation $\sigma(m_0) = \sigma(r | m_0)$

see e.g. M.H.R. Stanley et al., nature, 1996.

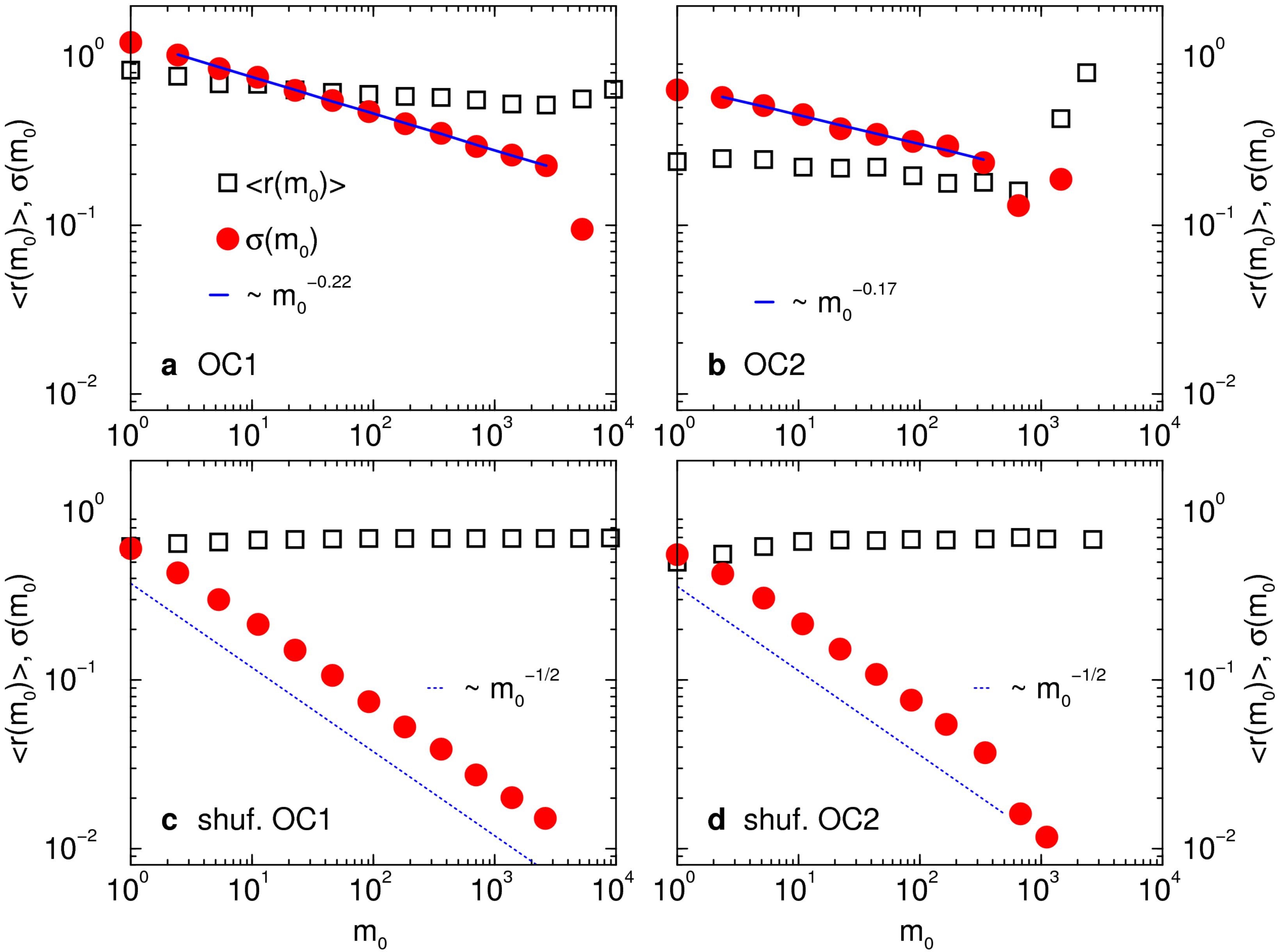
Analogy to other data, such as city growth

(1) The members of a community represent a population similar to the population of a country.

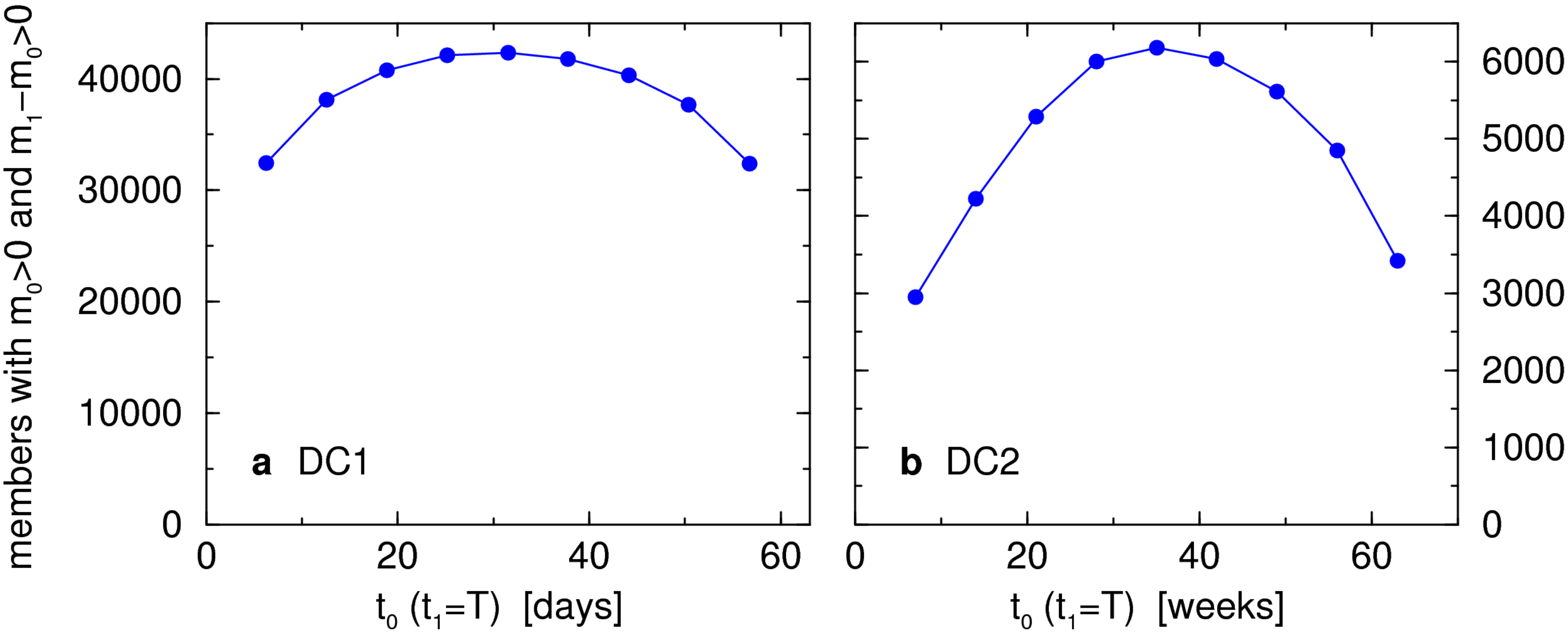
(2) The number of members fluctuates and typically grows analogous to the number of cities of a country.

(3) The activity or number of links of individuals fluctuates and grows similar to the size of cities.

Growth process: results



Optimal times



Growth process: results

$$\sigma(m_0) \sim m_0^{-\beta}$$

OC1: $\beta_{\mathrm{OC1}} = 0.22 \pm 0.01$
OC2: $\beta_{\mathrm{OC1}} = 0.17 \pm 0.03$
shuffled: $\beta_{\mathrm{rnd}} = 1/2$

Gibrat's law of proportionate growth

multiplicative process
to explain broad distributions (log-normal)

involves assumption: $\langle r(m_0) \rangle = \mathrm{const.}$
$\sigma(m_0) = \mathrm{const.}$

=> $\beta_{\mathrm{G}} = 0$

Temporal correlations

- shuffling destroys temporal correlations, leading to $\beta_{\mathrm{rnd}} = 1/2$

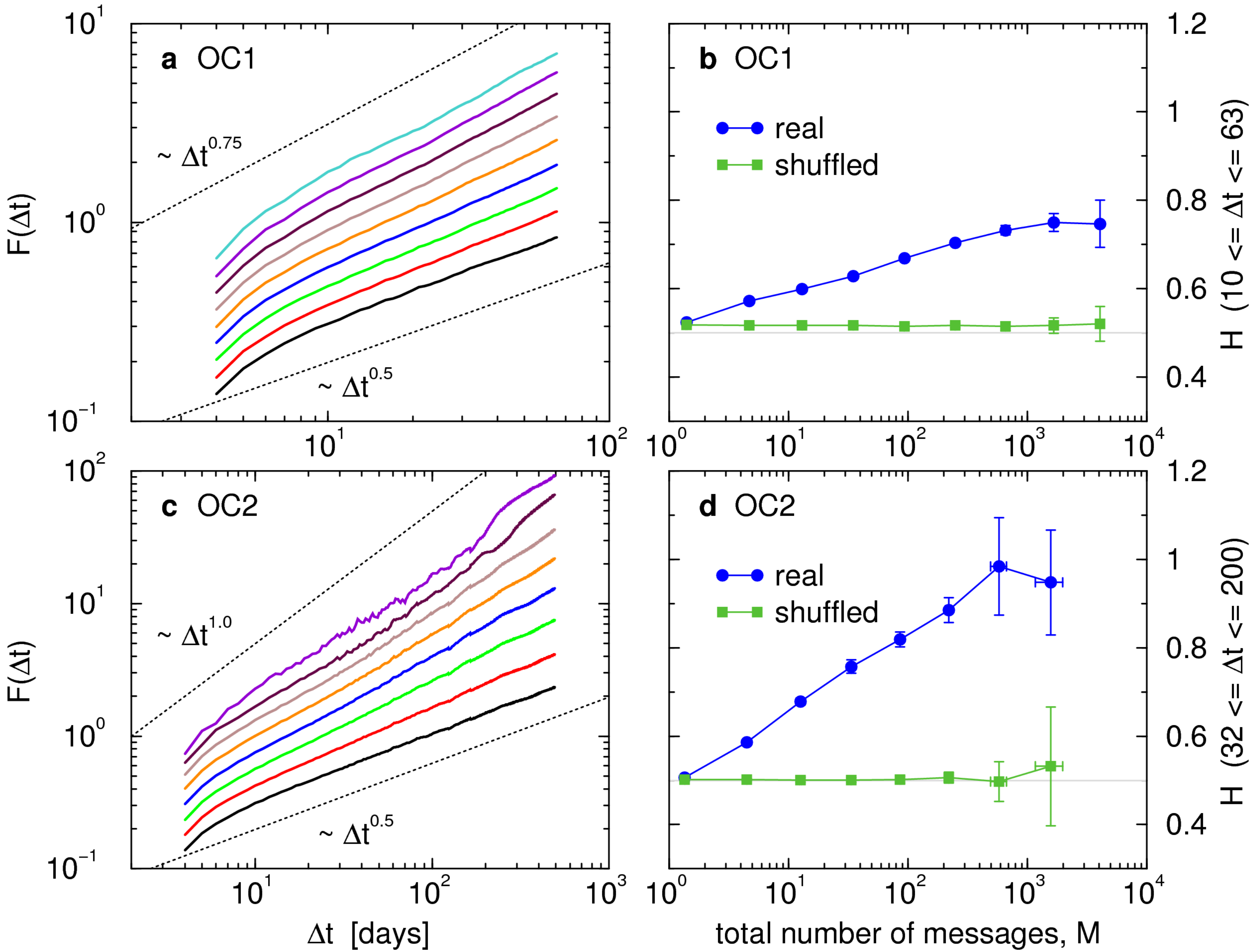
- this suggests $\beta \approx 0.2$ might be due to temporal correlations

- we use Detrended Fluctuation Analysis (DFA) to quantify long-term correlations in the activity (messages per day): $\mu(t)$

fluctuation function: $F(\Delta t) \sim (\Delta t)^H$

$1/2 < H < 1$ => ltc

Temporal correlations: results



Missing link

derivation leads to:

$$\boxed{\beta = 1 - H}$$

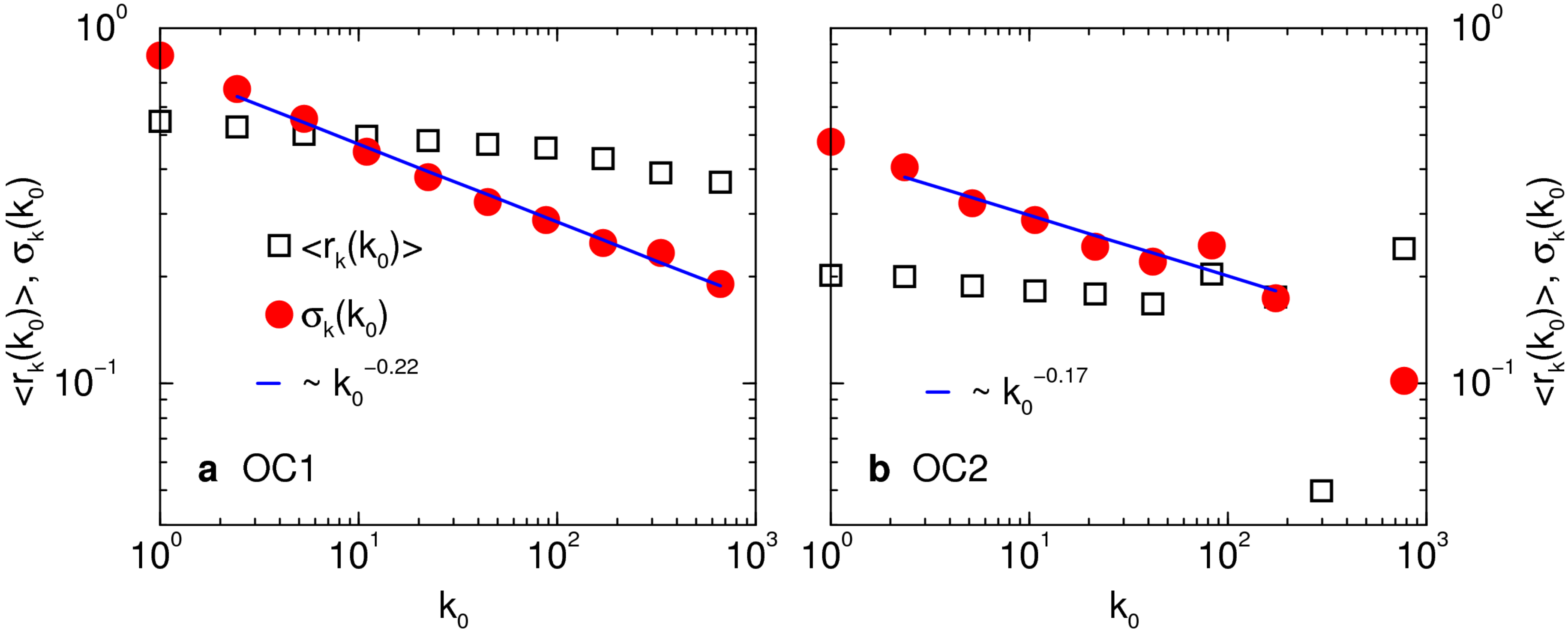
accordingly:

$$\beta \approx 0.2 \quad \Rightarrow \quad H \approx 0.8 \qquad \text{OCs}$$

$$\beta_{\mathrm{rnd}} = 1/2 \quad \Rightarrow \quad H_{\mathrm{rnd}} = 1/2 \qquad \text{shuffled}$$

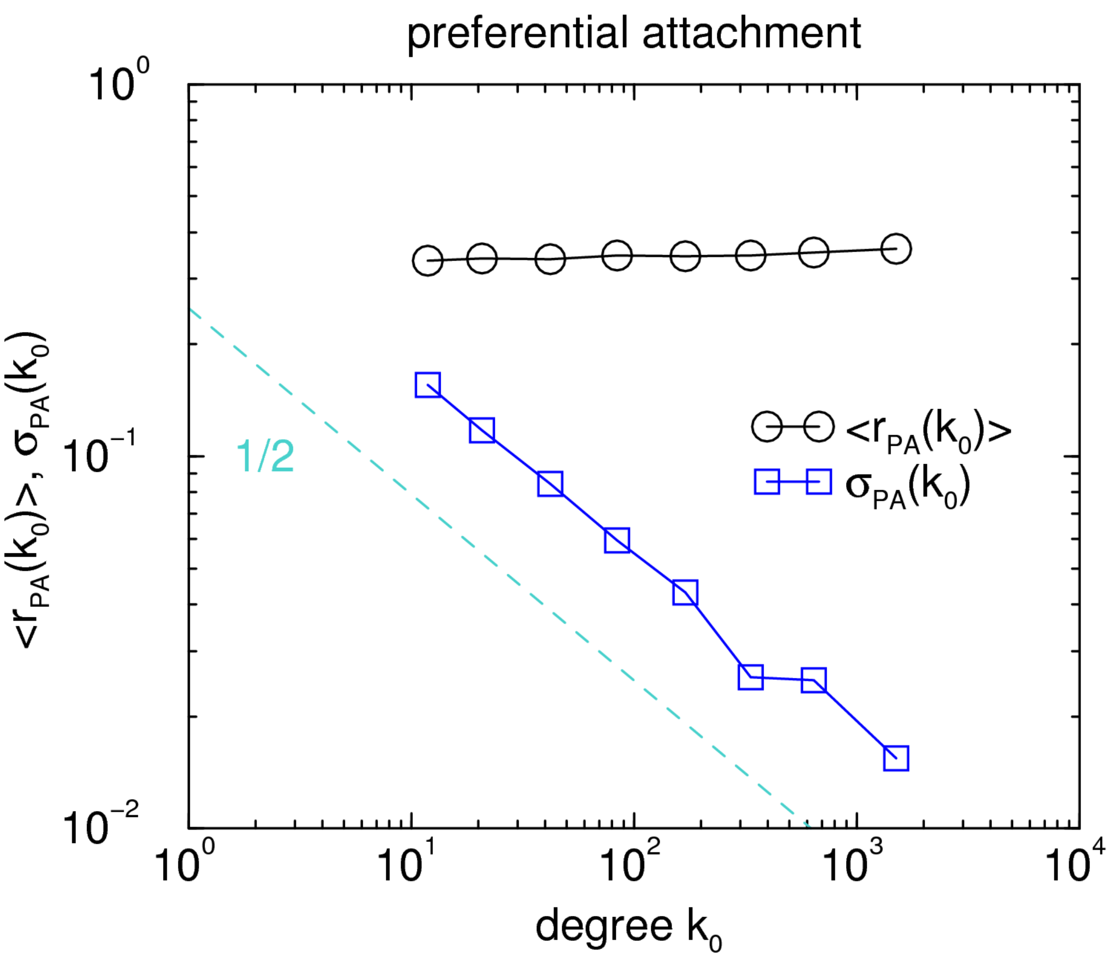
$$\beta_{\mathrm{G}} = 0 \quad \Rightarrow \quad H_{\mathrm{G}} = 1 \qquad \text{Gibrat's law}$$

Growth process: out-degree



see also: Maillart T, et al., arXiv 0807.0014, 2008

Growth process: preferential attachment



see also: Barabasi AL and Albert R, Science 286, 1999

Conclusions

1. scaling in growth of number of messages or out-degree implies that active members are better predictable than less active ones

2. human activity sending messages is long-term correlated

3. scaling in growth is due to long-term correlations

$$\sigma(m_0) \sim m_0^{-\beta}$$

=> this may also be the case for other data

Thank you for your attention.

paper submitted to PNAS: Rybski D, *et al.*