POTSDAM INSTITUTE FOR CLIMATE IMPACT RESEARCH

Proximity based city growth

Diego Rybski,

Anselmo García Cantú Ros,
Jürgen P. Kropp

SOE 18.4
DPG Frühjahrstagung 2012
Berlin, H 0110
29.3.2012 – 11:00

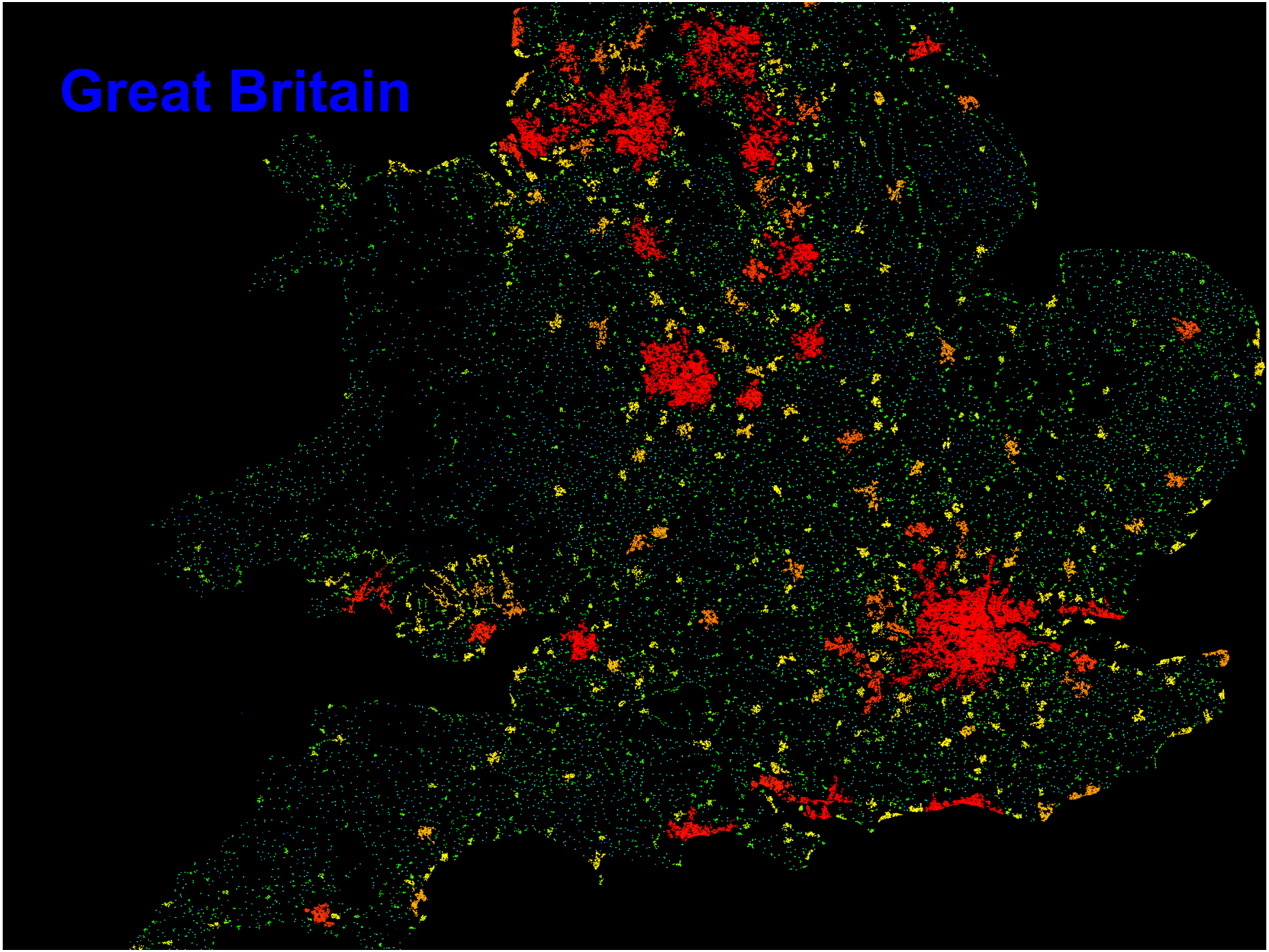
Motivation

B.J.L. Berry & A. Okulicz-Kozaryn 2011:

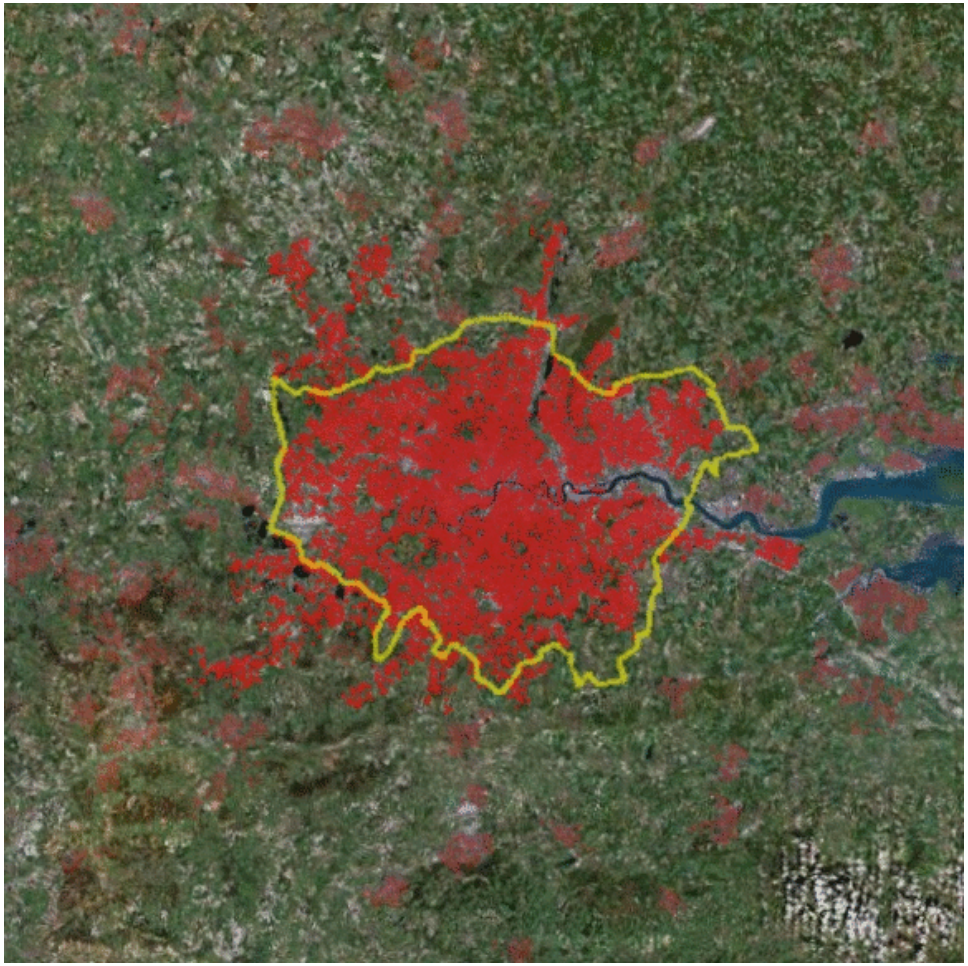
- identify different epochs of research on city size distributions
  + An interesting empirical regularity (1913-1936)
  + Zipf's Law (1939-...)
  + Building a body of theory
  + A stochastic steady-state
  + Contemporary contentions

- raise three questions:
  1. What are the appropriate units of observation?
  2. Do the growth rates obey Gibrat's law?
  3. Does Zipf's Law apply in the upper tail?

1.+2. Rozenfeld et al., PNAS, 2008
1.+3. Rozenfeld et al., AER, 2011
City Clustering Algorithm (CCA)


Great Britain

Greater London



First Law of Geography

"*Everything is related to everything else, but near things are more related than distant things.*"

(W.R. Tobler 1970)

Gravitation Concepts

G.A.P. Carrothers (1956): “An Historical Review of the Gravity and Potential Concepts of Human Interaction”

W.J. Reilly's law (1931): breaking point of boundary of equal attraction

D.L. Huff's law (1963/64): shopper attraction

P. Pöyhönen (1963): “A Tentative Model for the Volume of Trade between Countries”

[F. Simini (2012): “A universal model for mobility and migration patterns”]

… we reexamine the gravitation concept

Approach

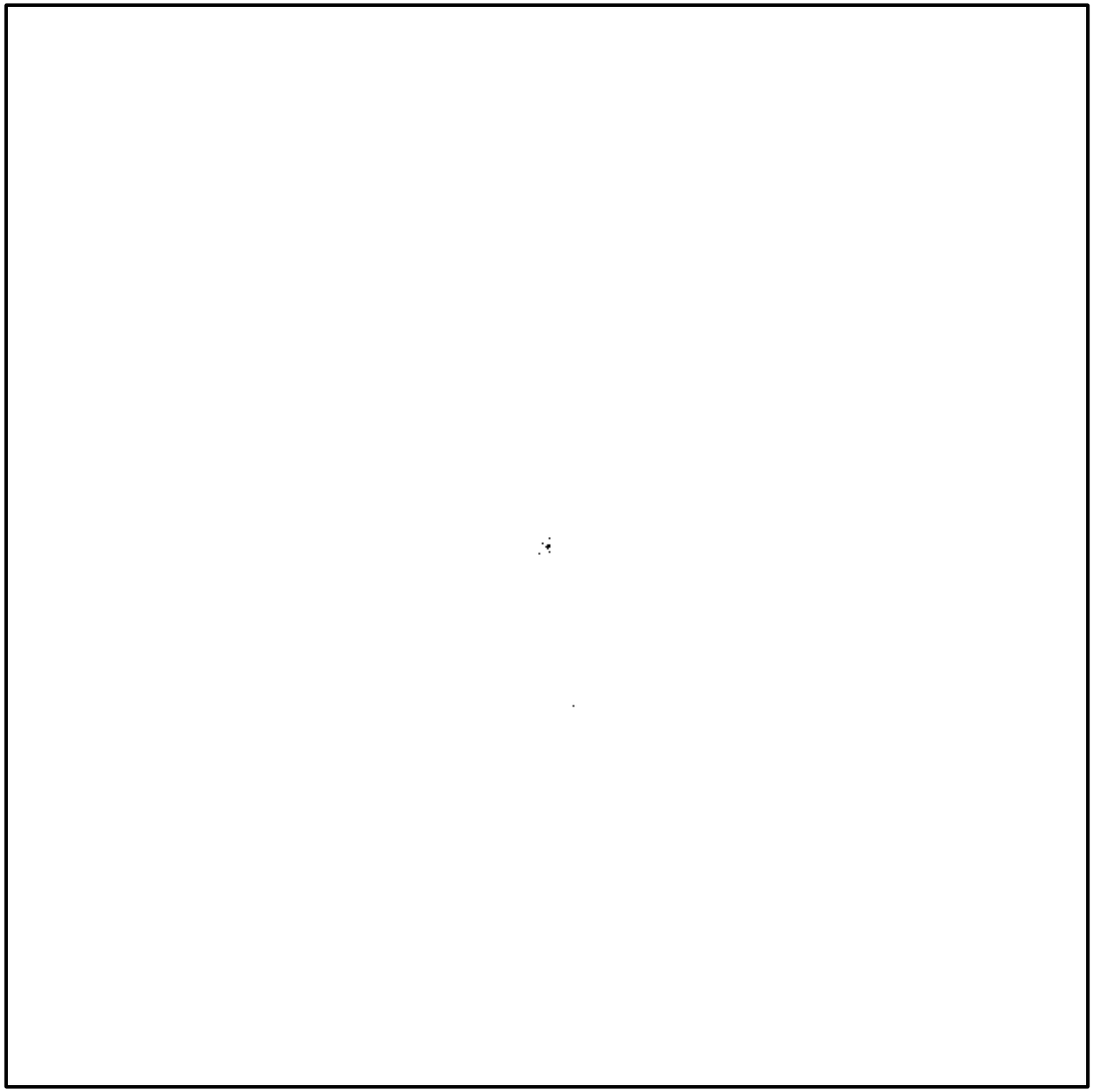
The cells of a grid with the coordinates i can either be occupied ($w_i = 1$) or empty ($w_i = 0$). Iteratively each site is tested for being populated. Therefore, the probability to become populated is given by

$$p_i = C \frac{\sum_{j \neq i} w_j d_{i,j}^{-\gamma}}{\sum_{j \neq i} d_{i,j}^{-\gamma}} \quad , \tag{1}$$

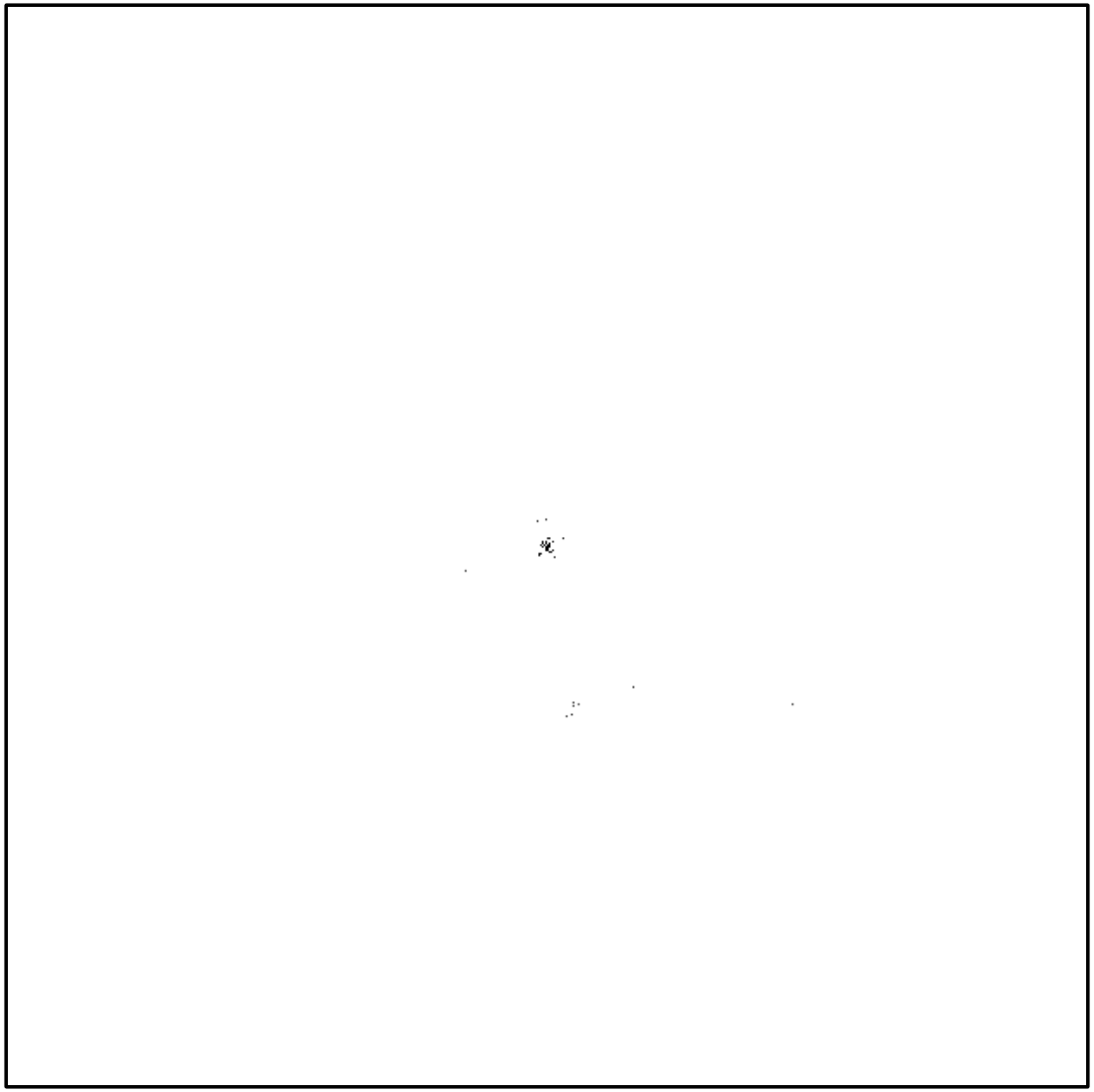
where $d_{i,j}$ is the distance between the points i and j. The index j runs over all sites with $w_j = 0$, i.e. already populated sites are not further considered. Finally, the probabilities are normalized according to $\max(p_i) = 1$.

Example

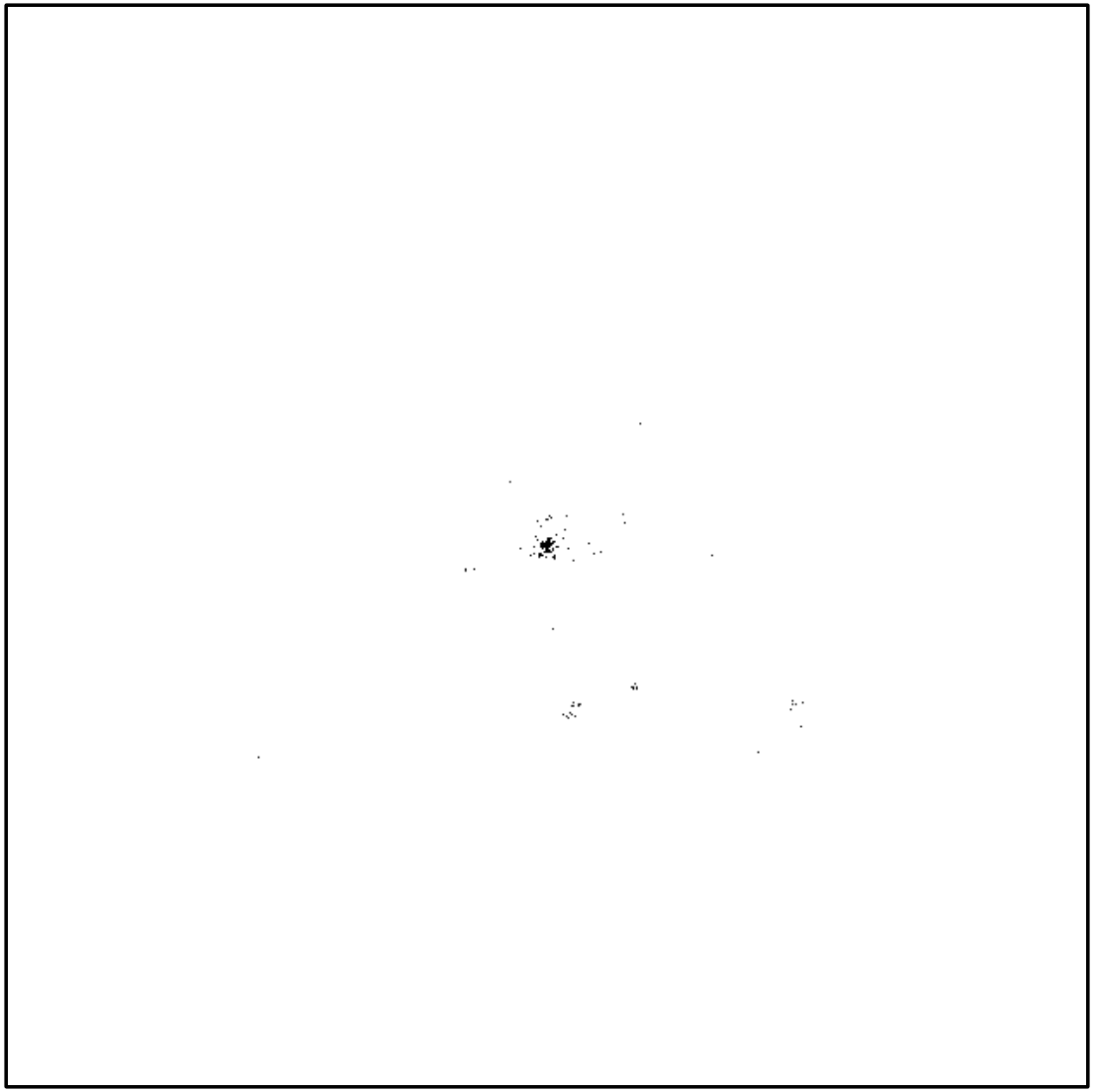
630 x 630
γ=2.5



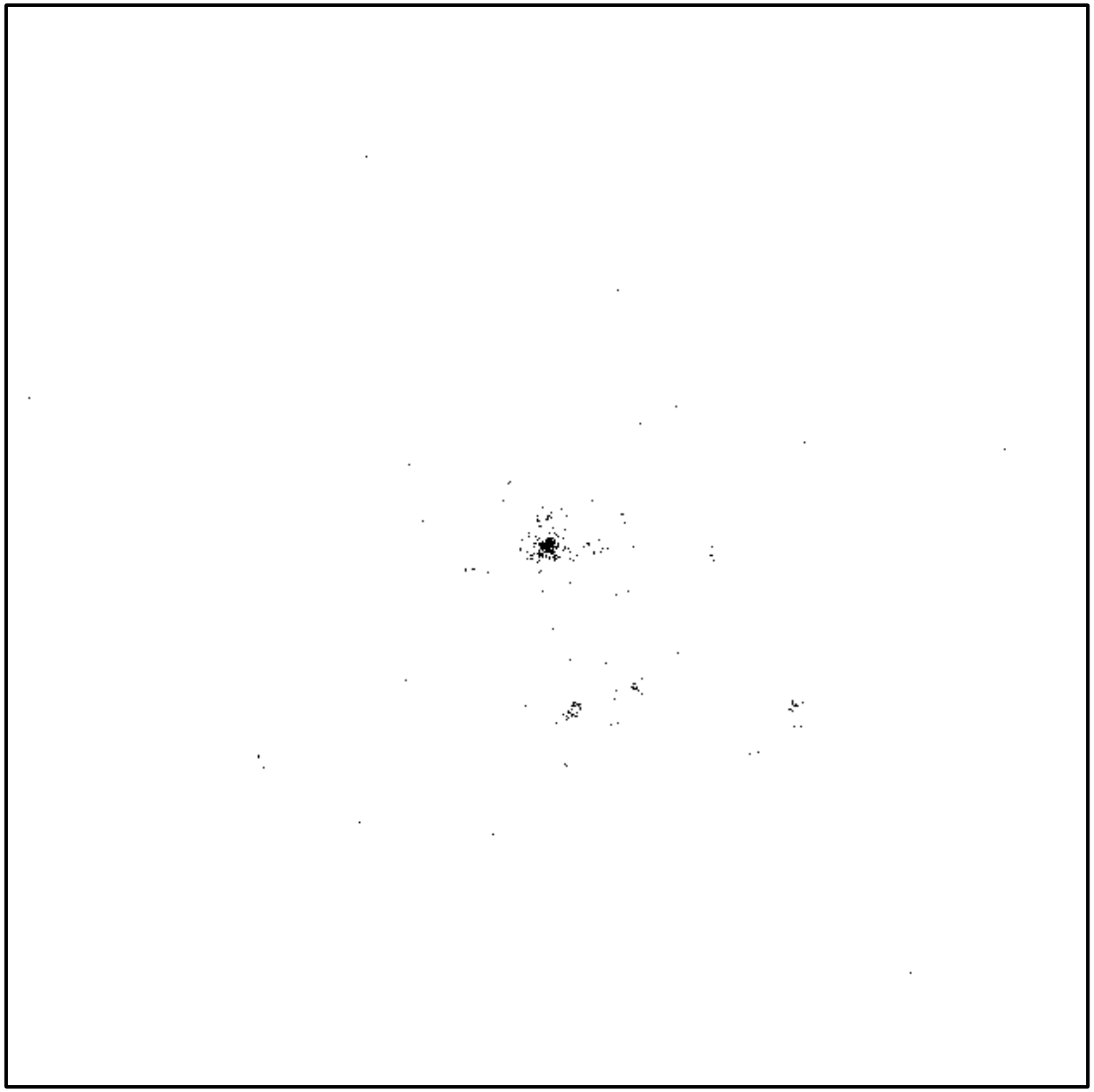
Example



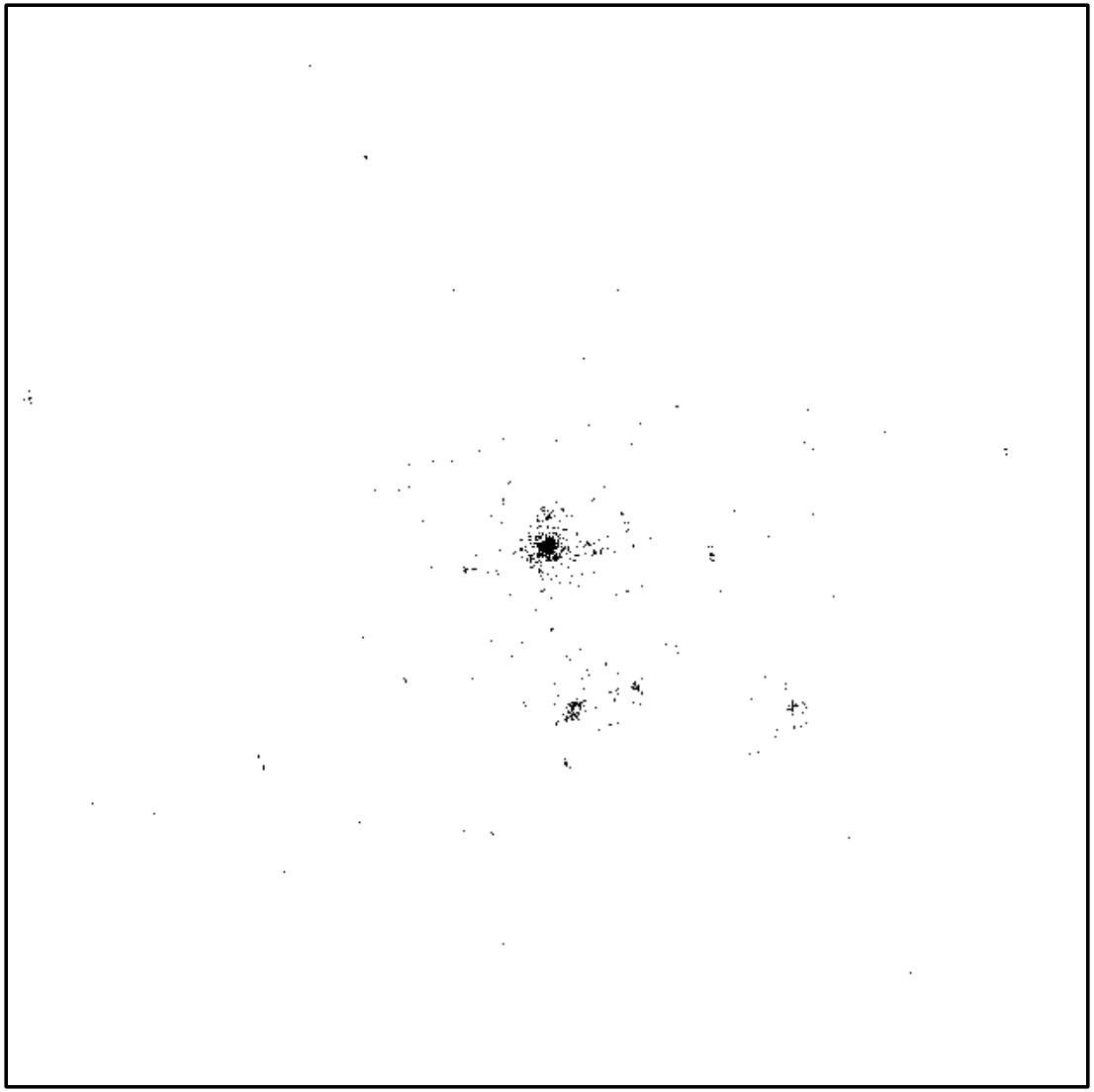
Example



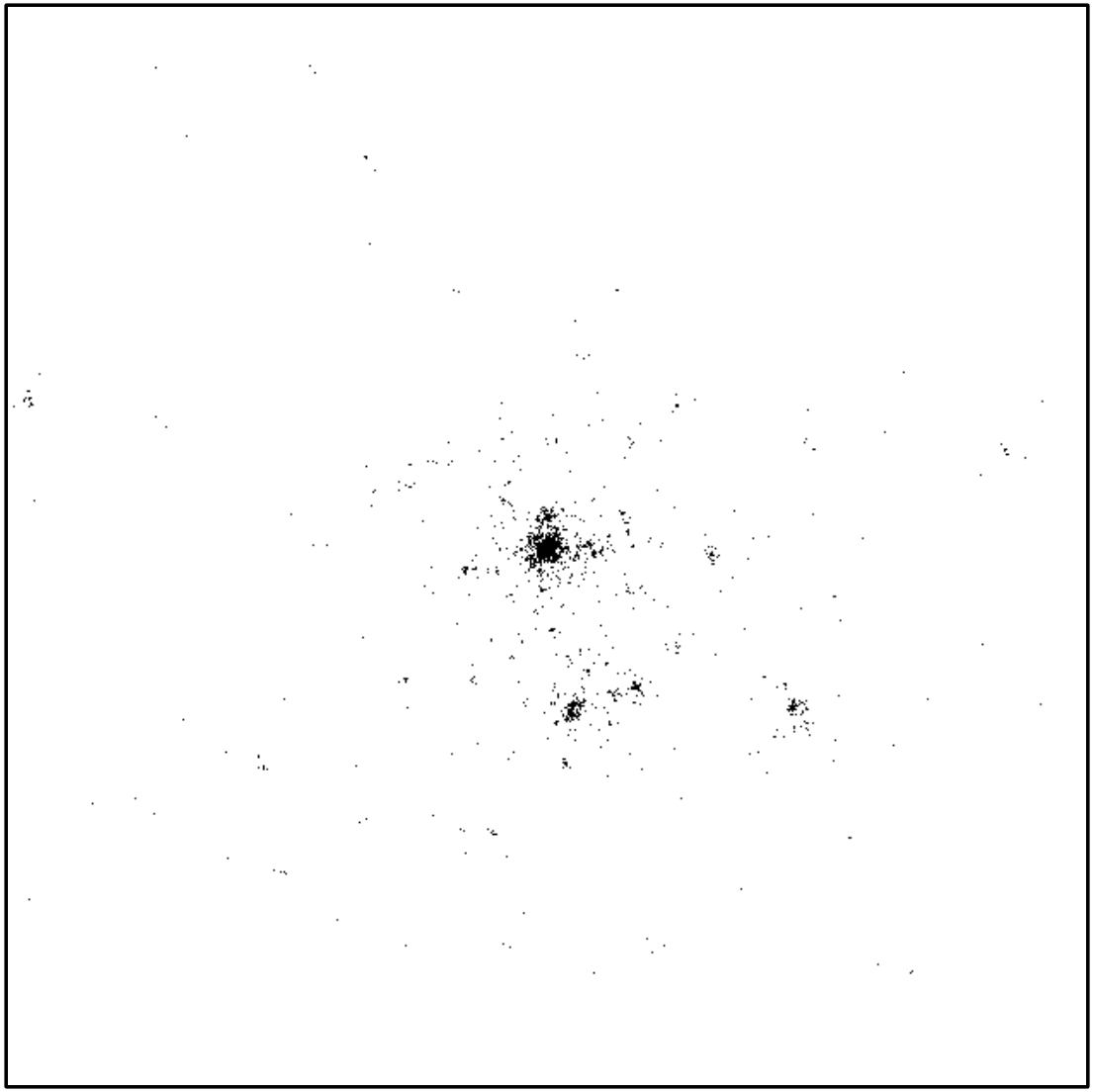
Example



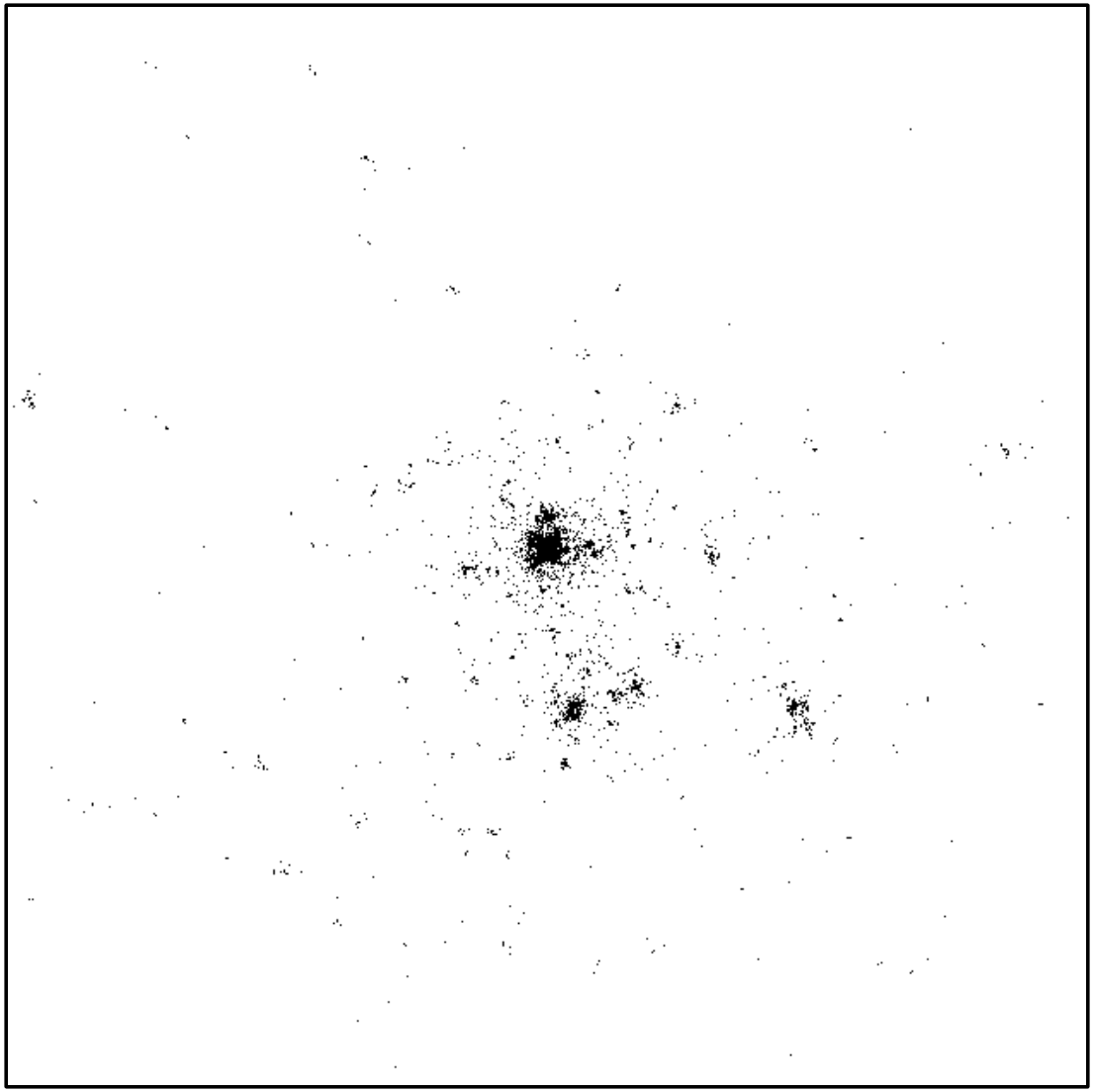
Example



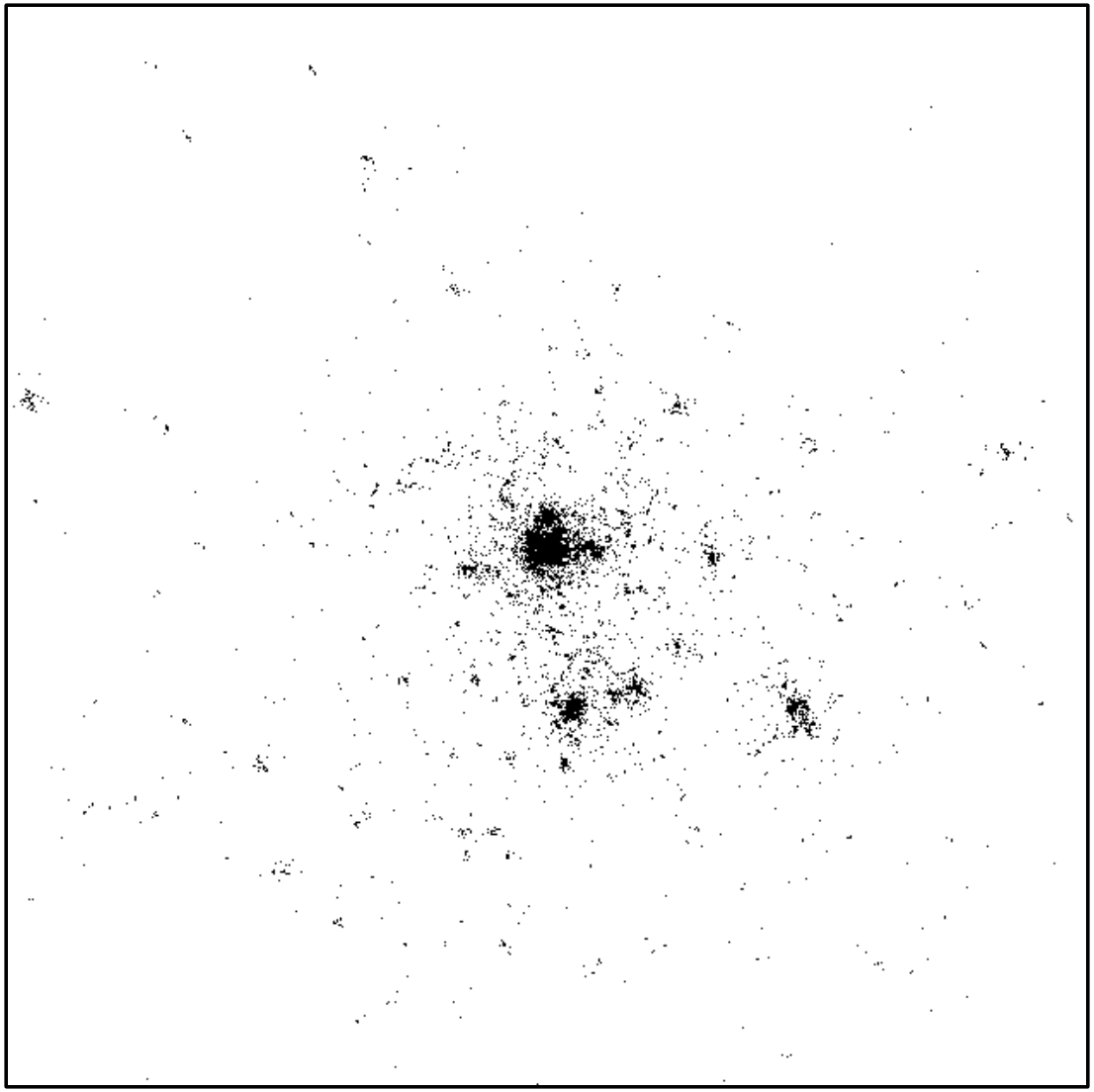
Example



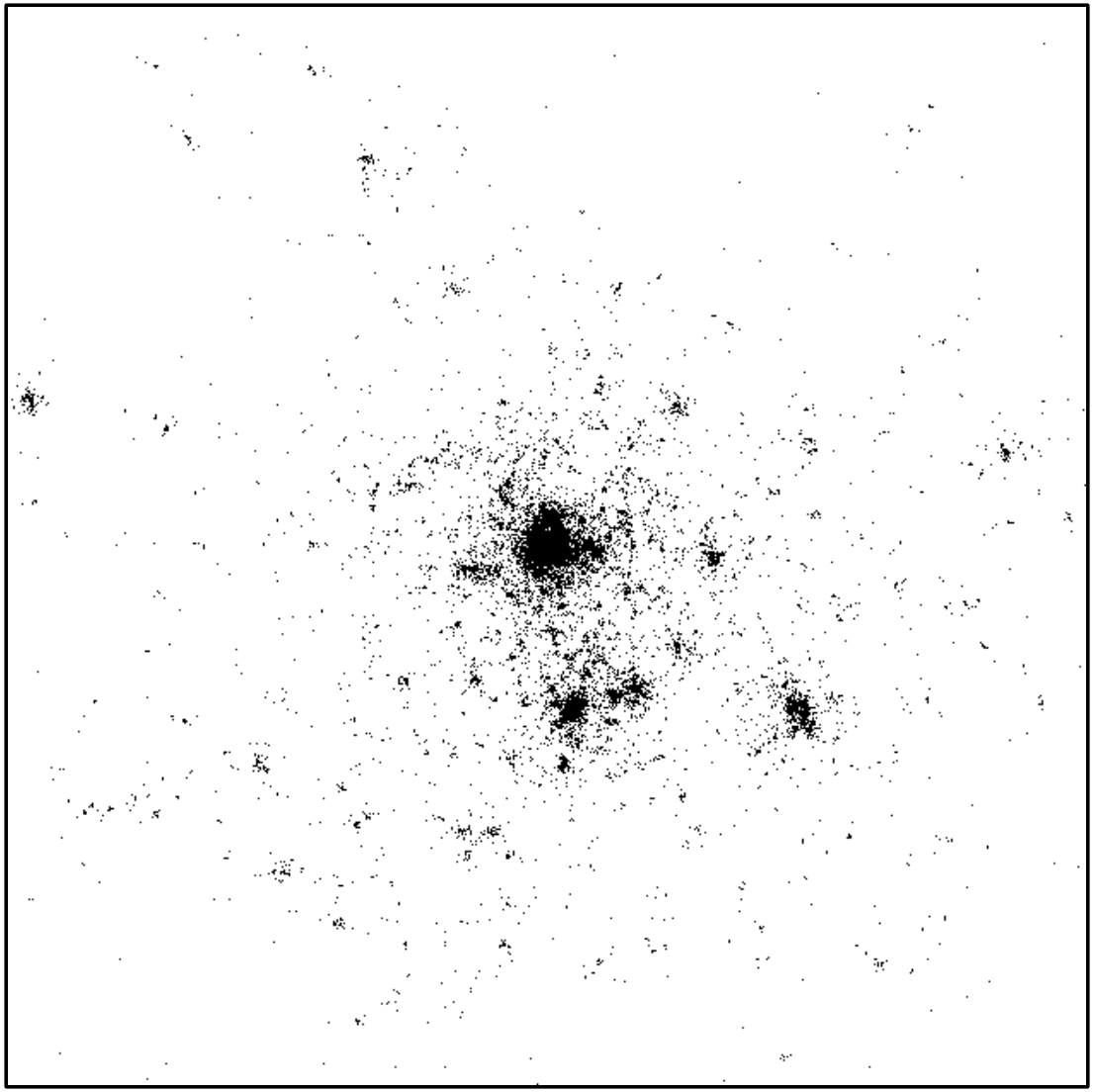
Example



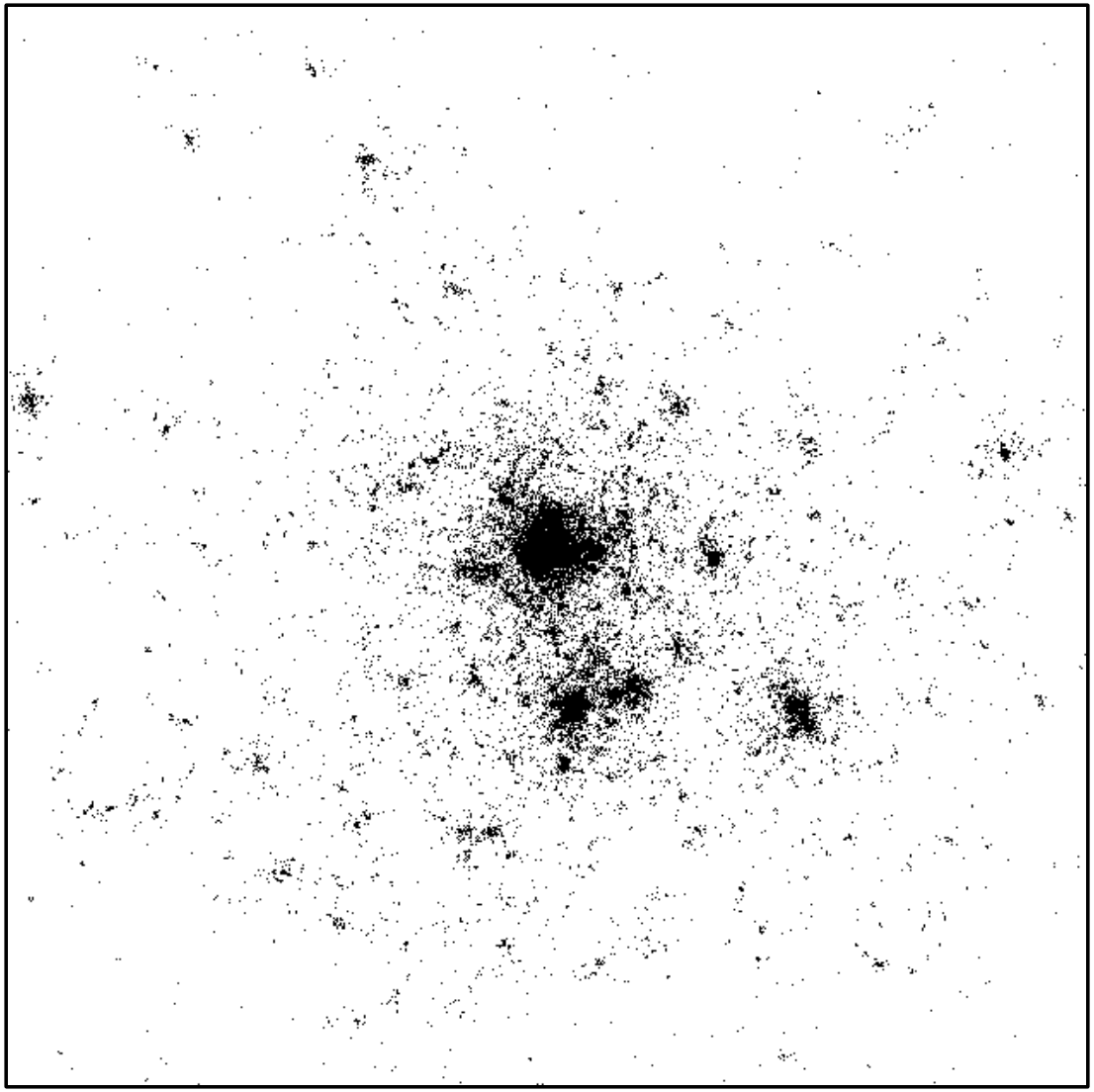
Example



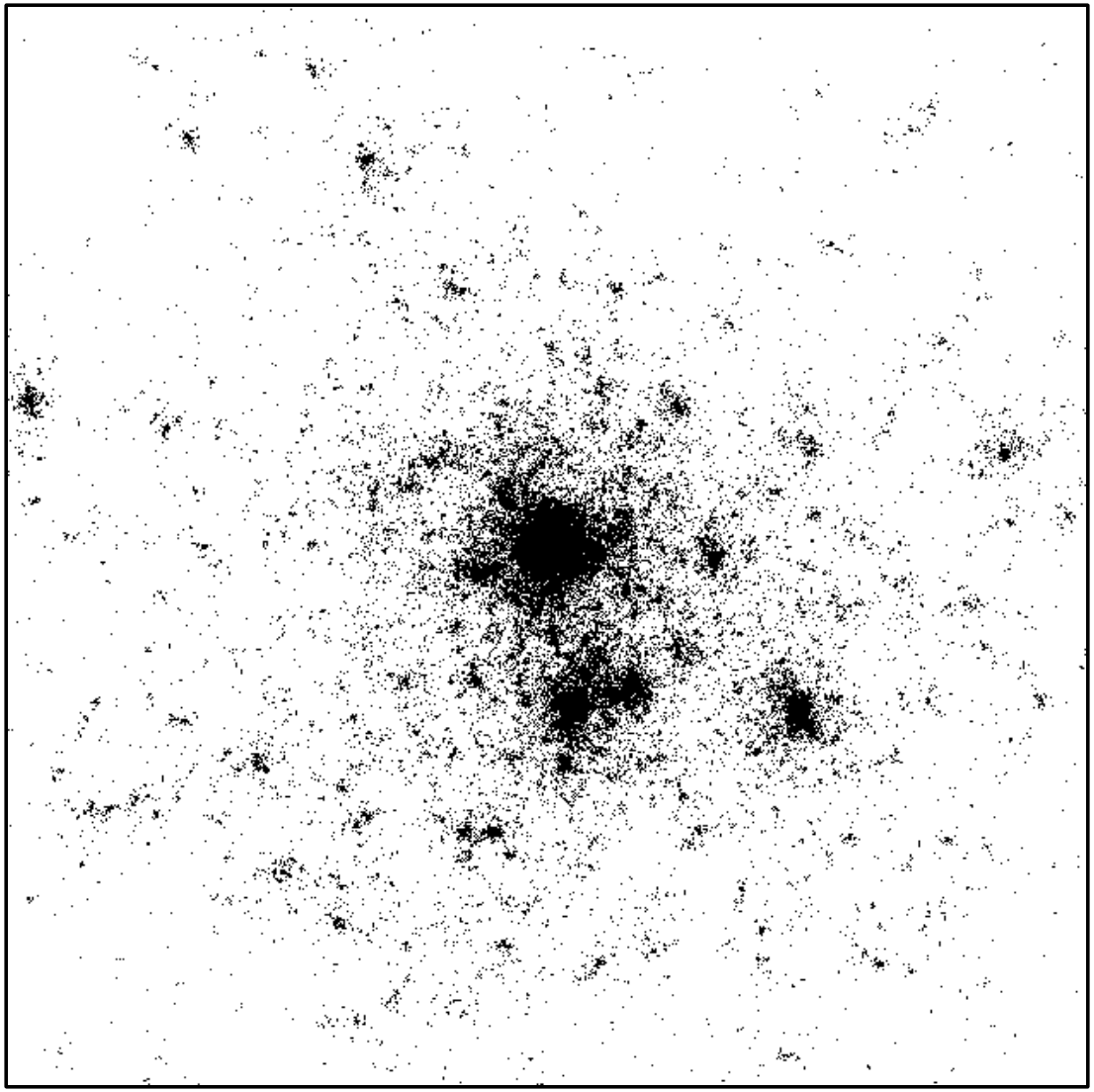
Example



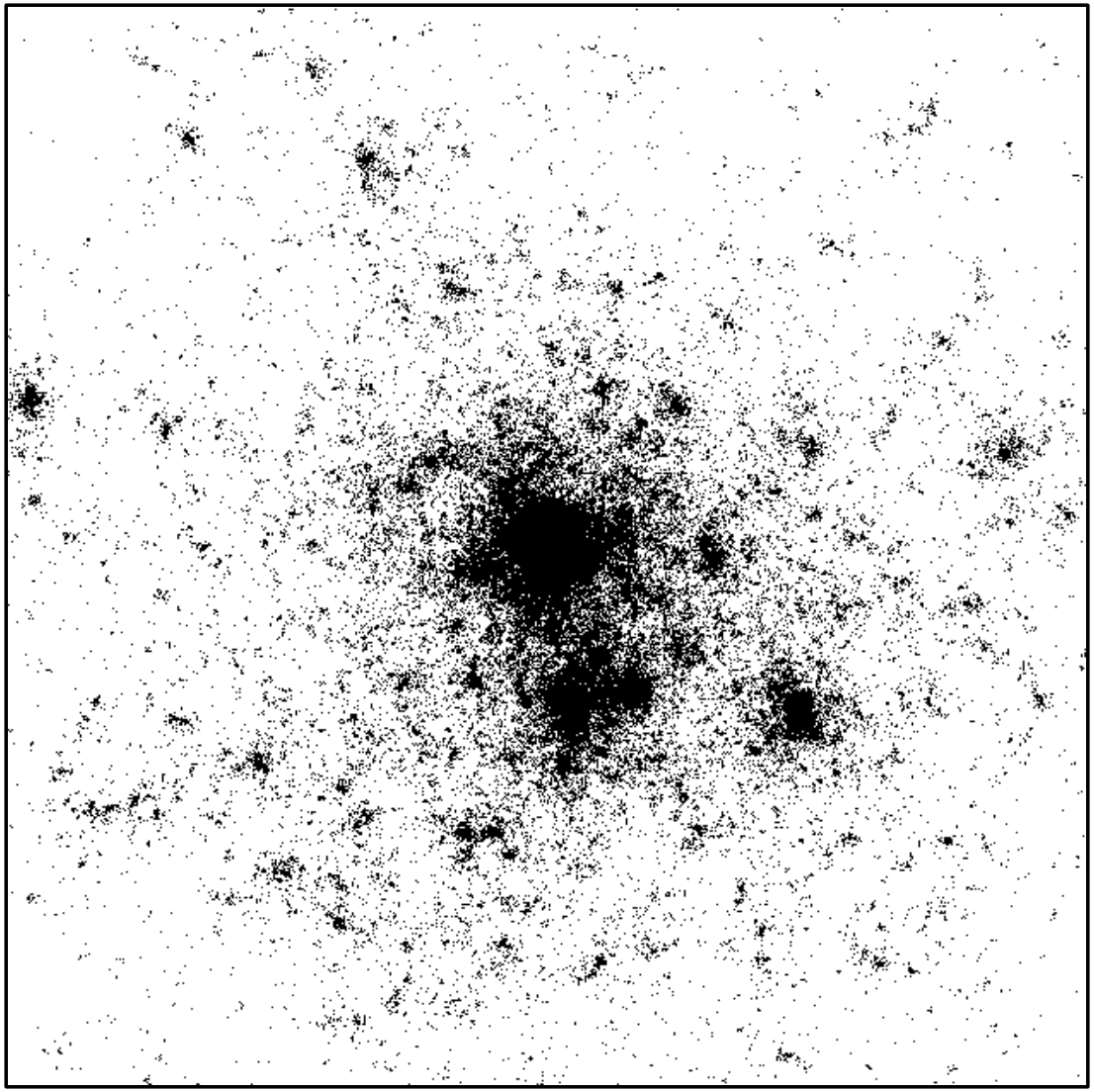
Example



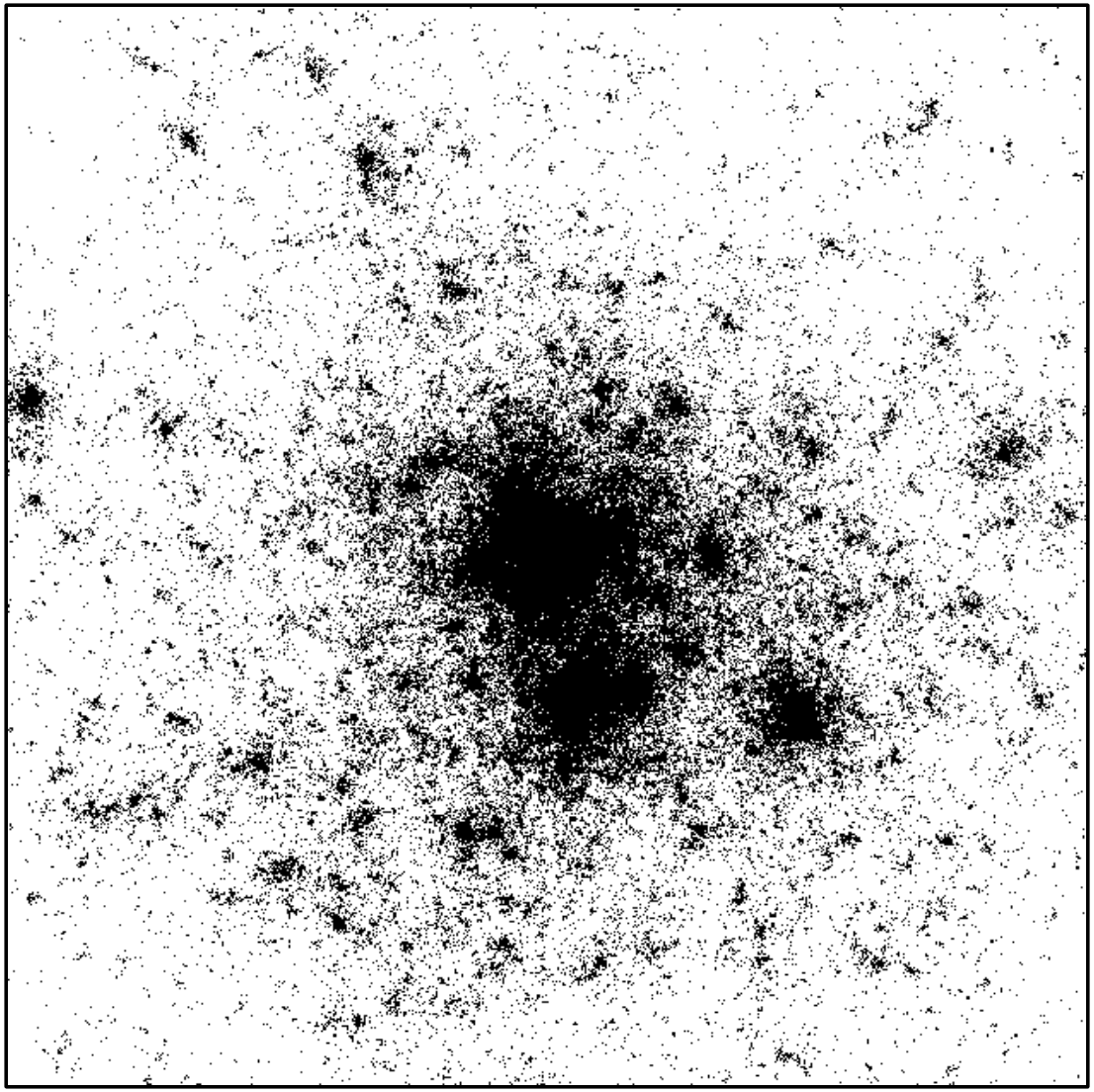
Example



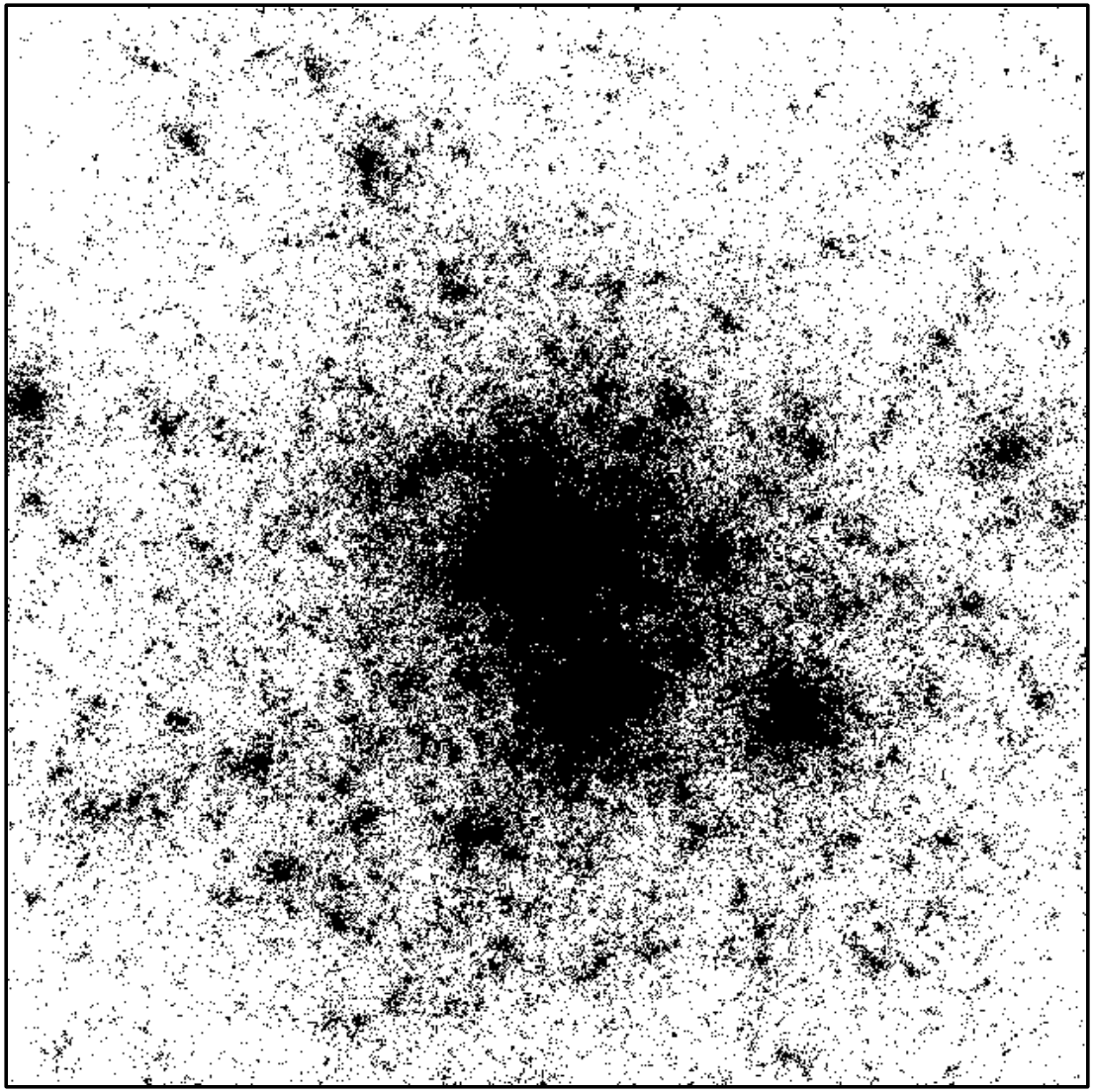
Example



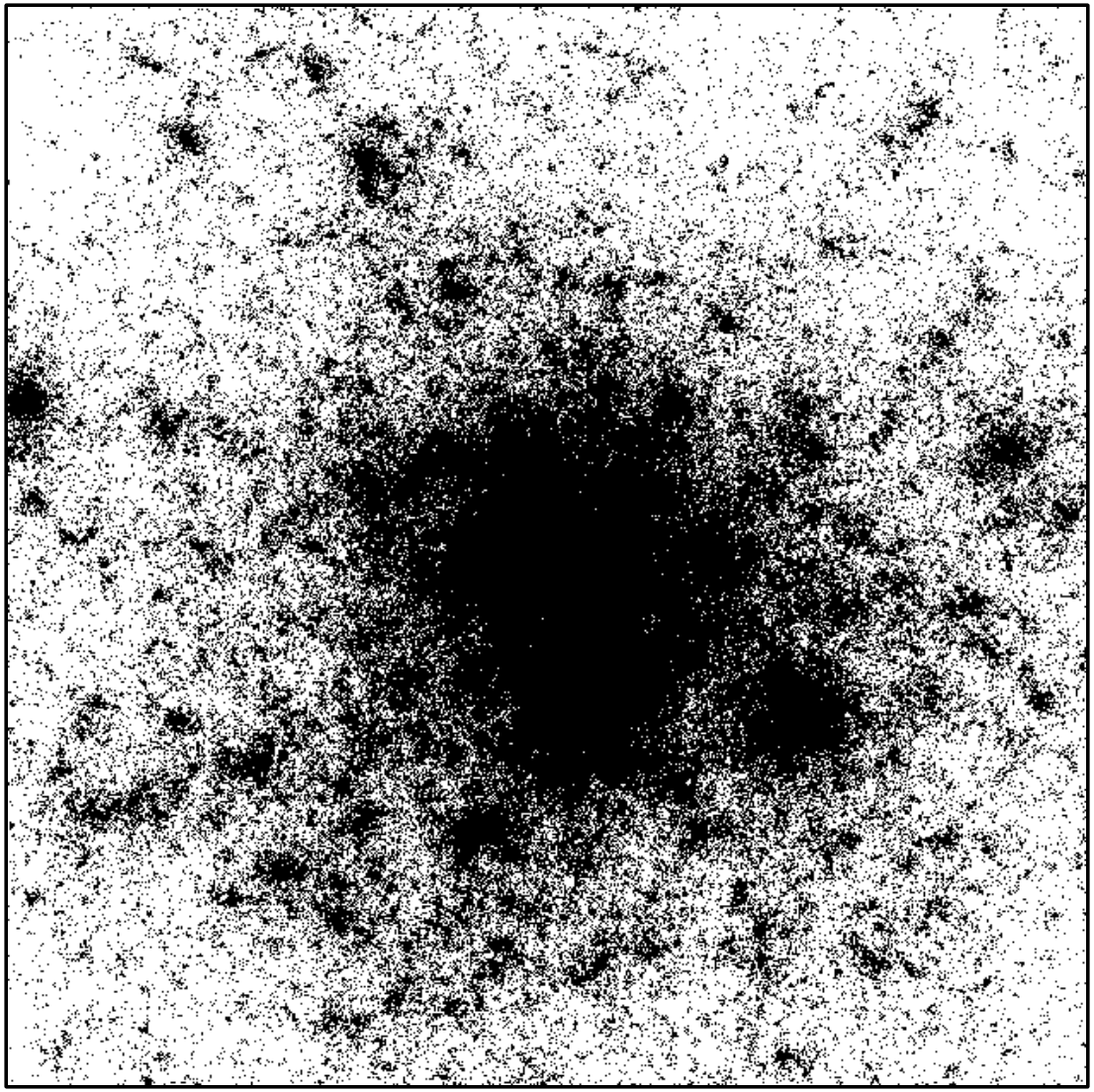
Example



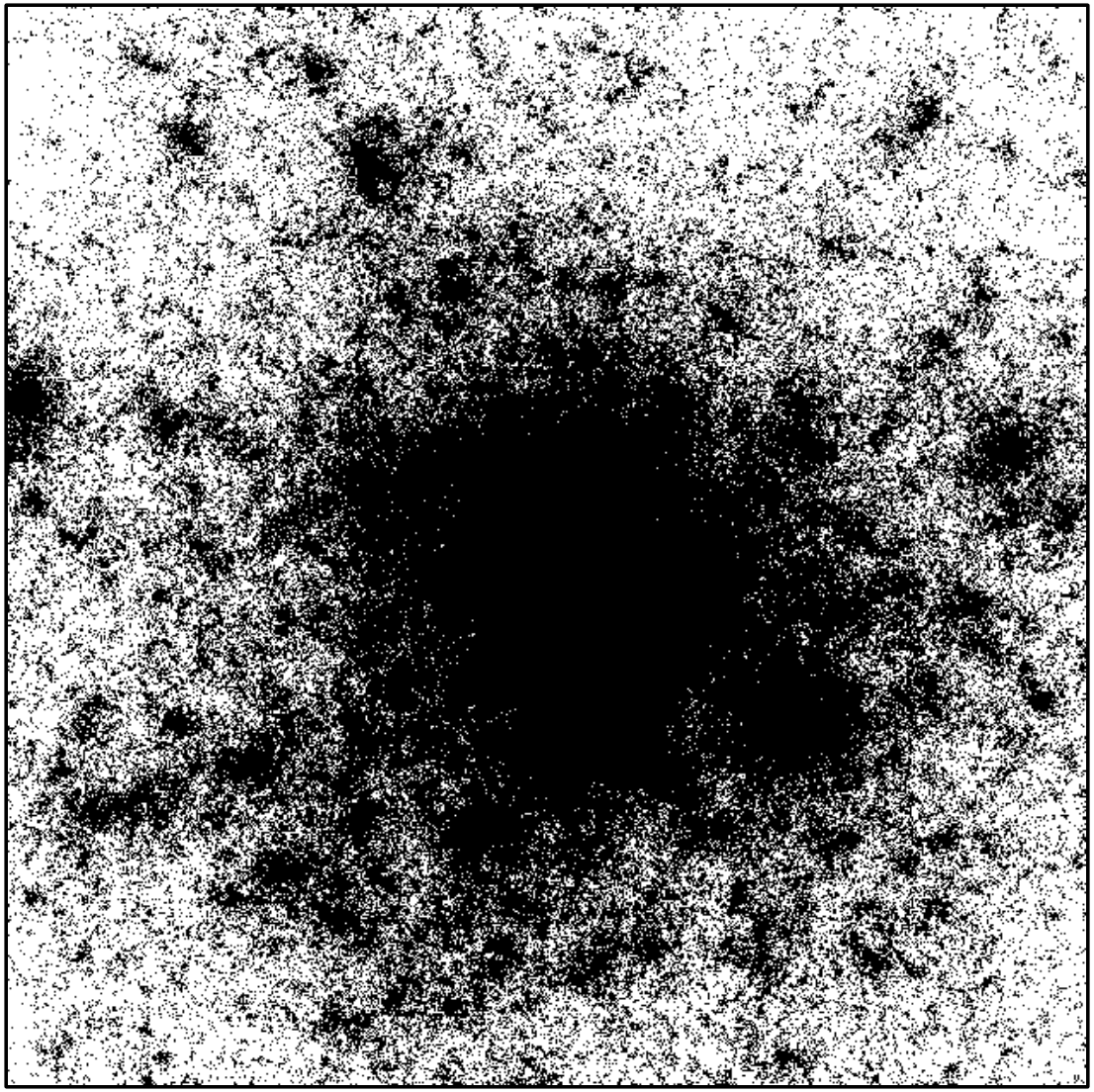
Example



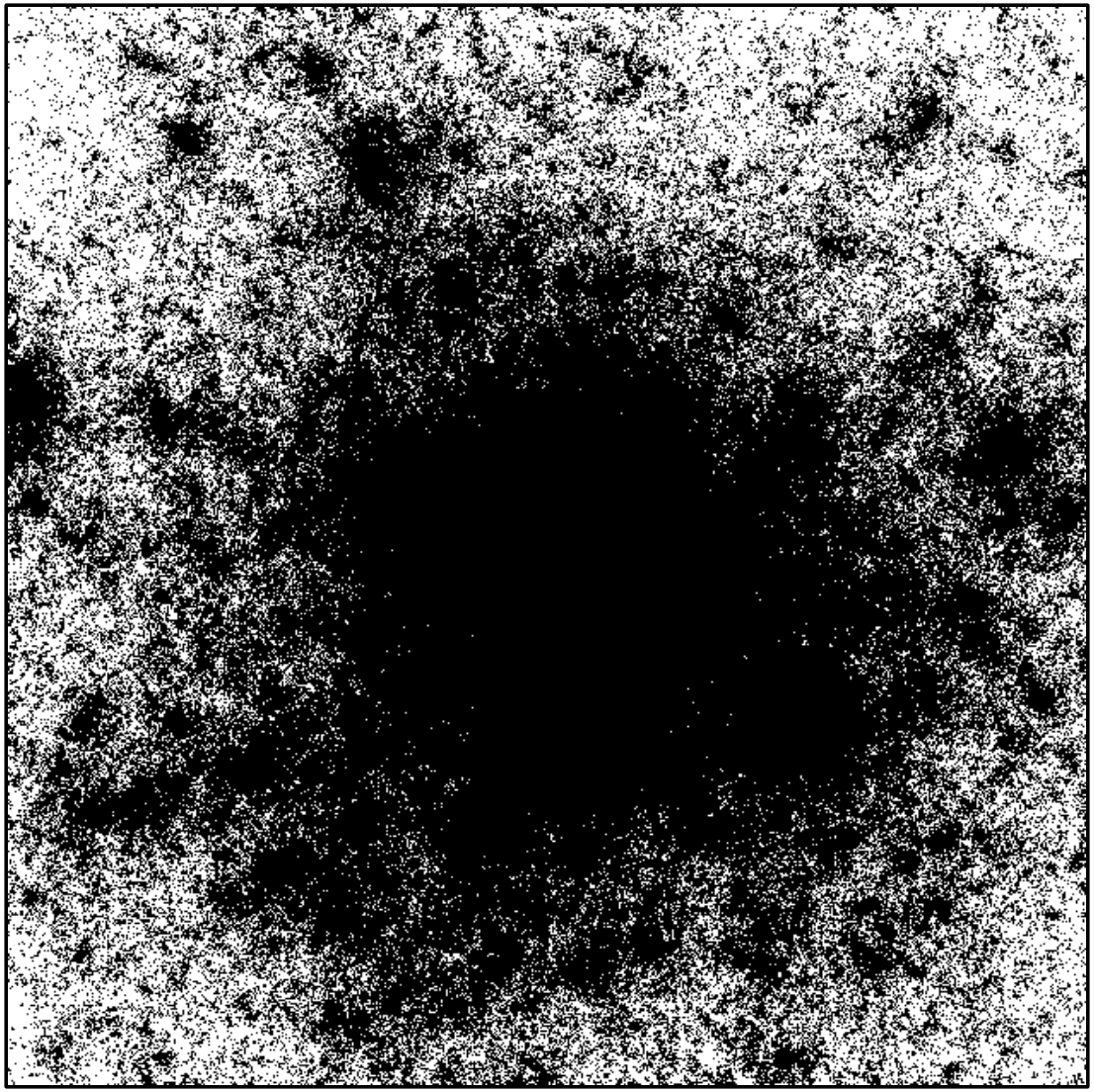
Example



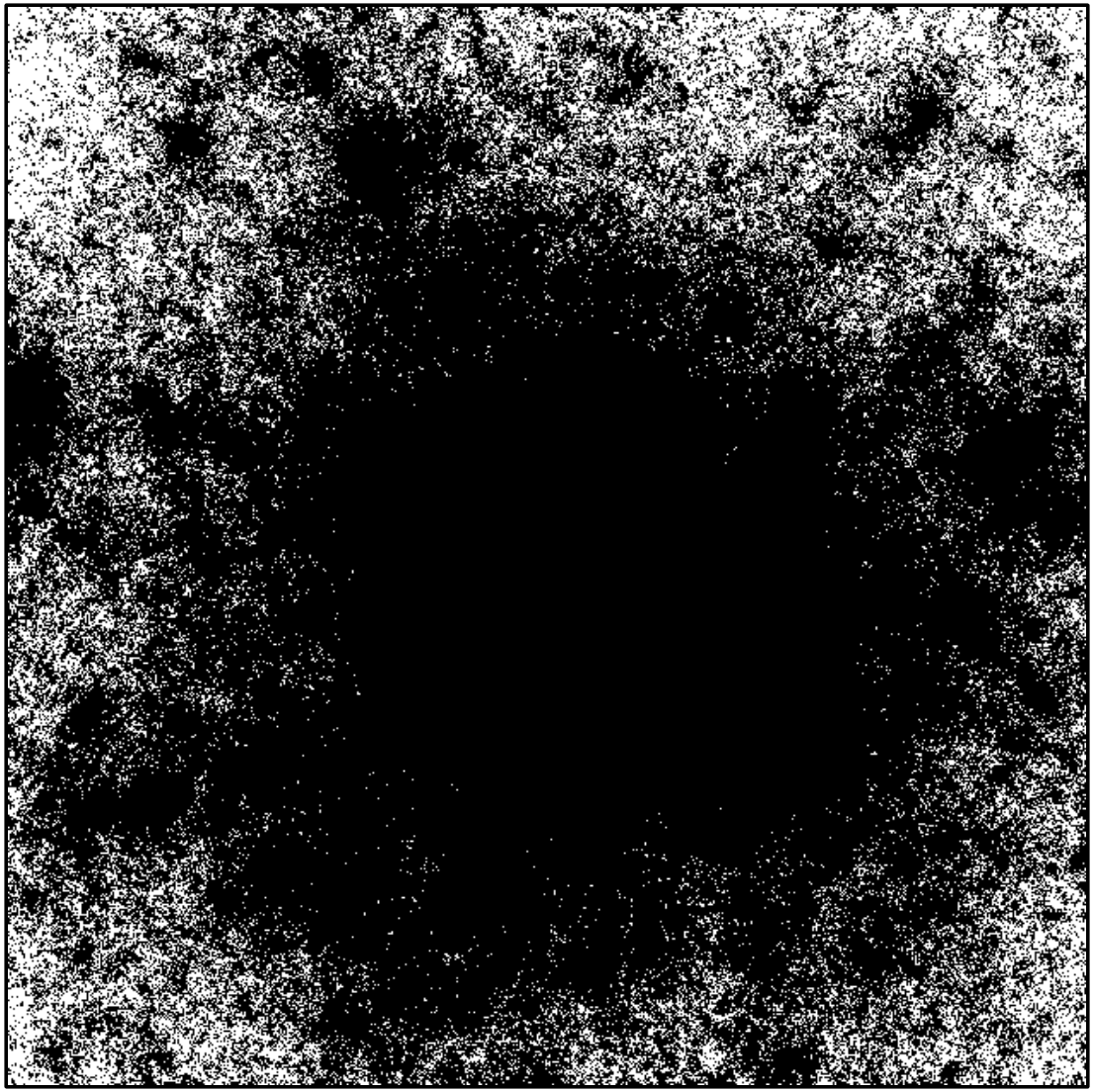
Example



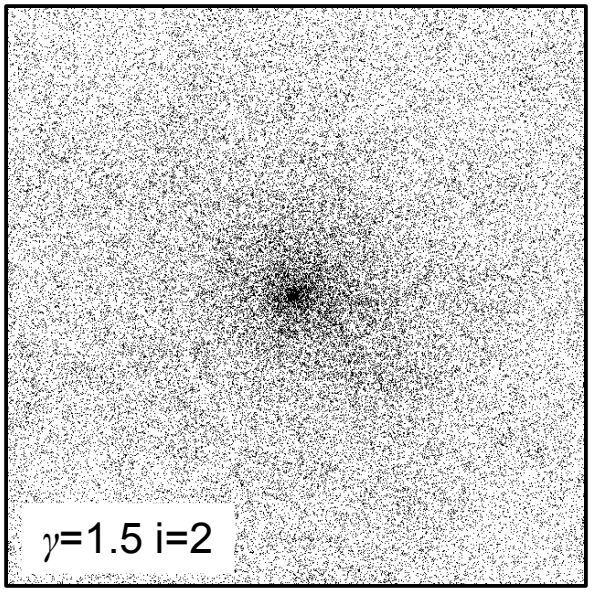
Example

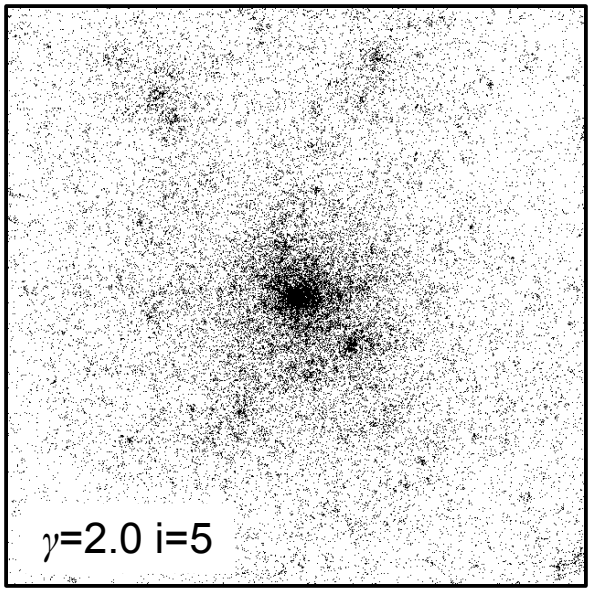


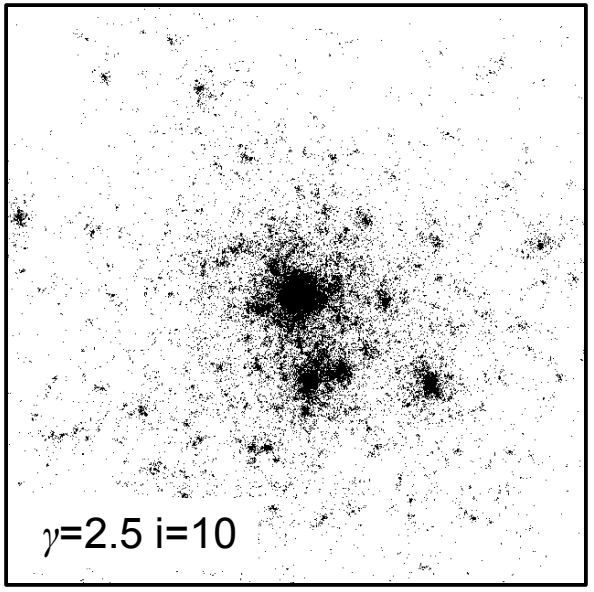
Example … end

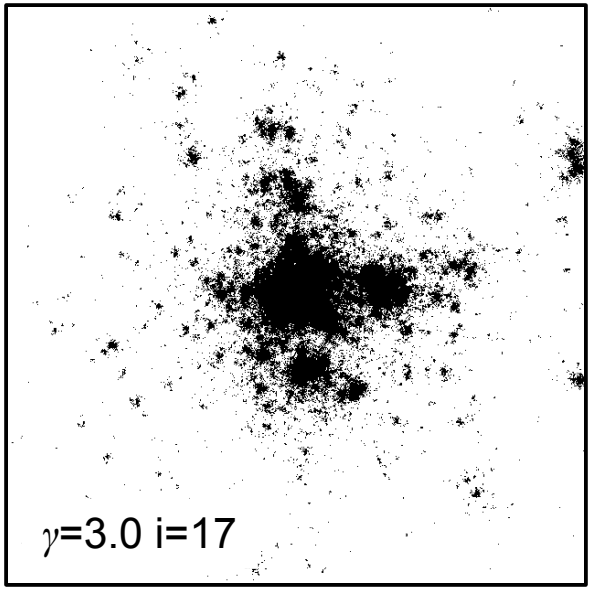


Exponent







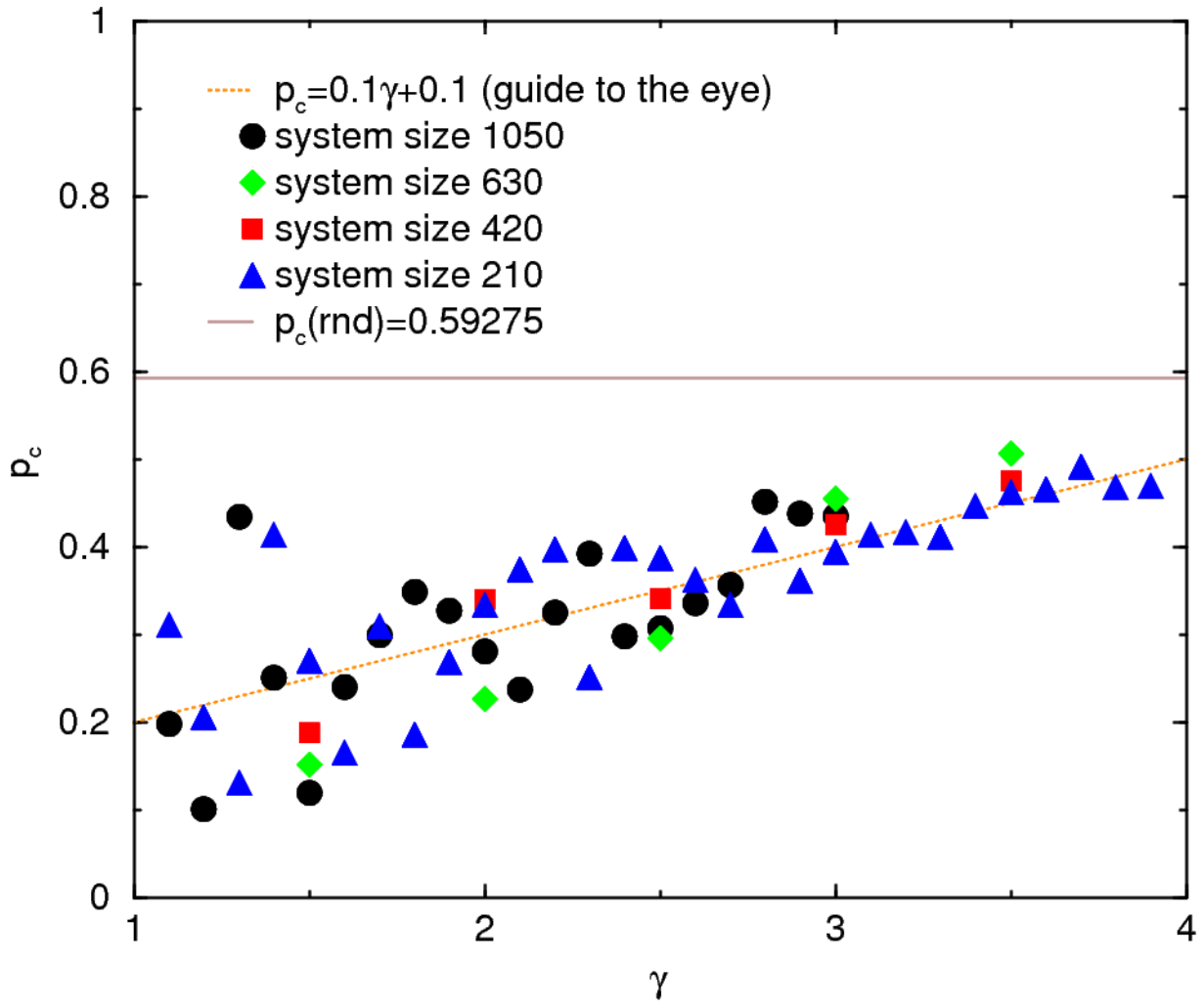


- small exponent fills faster

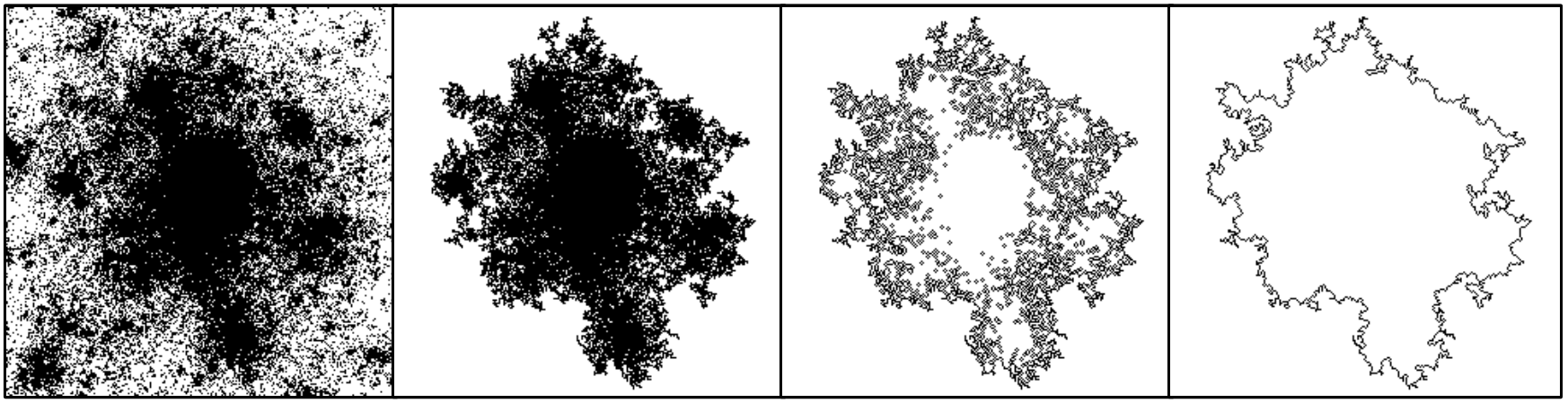
- large exponent more compact

Percolation transition

maximum in the number of clusters



Boundary



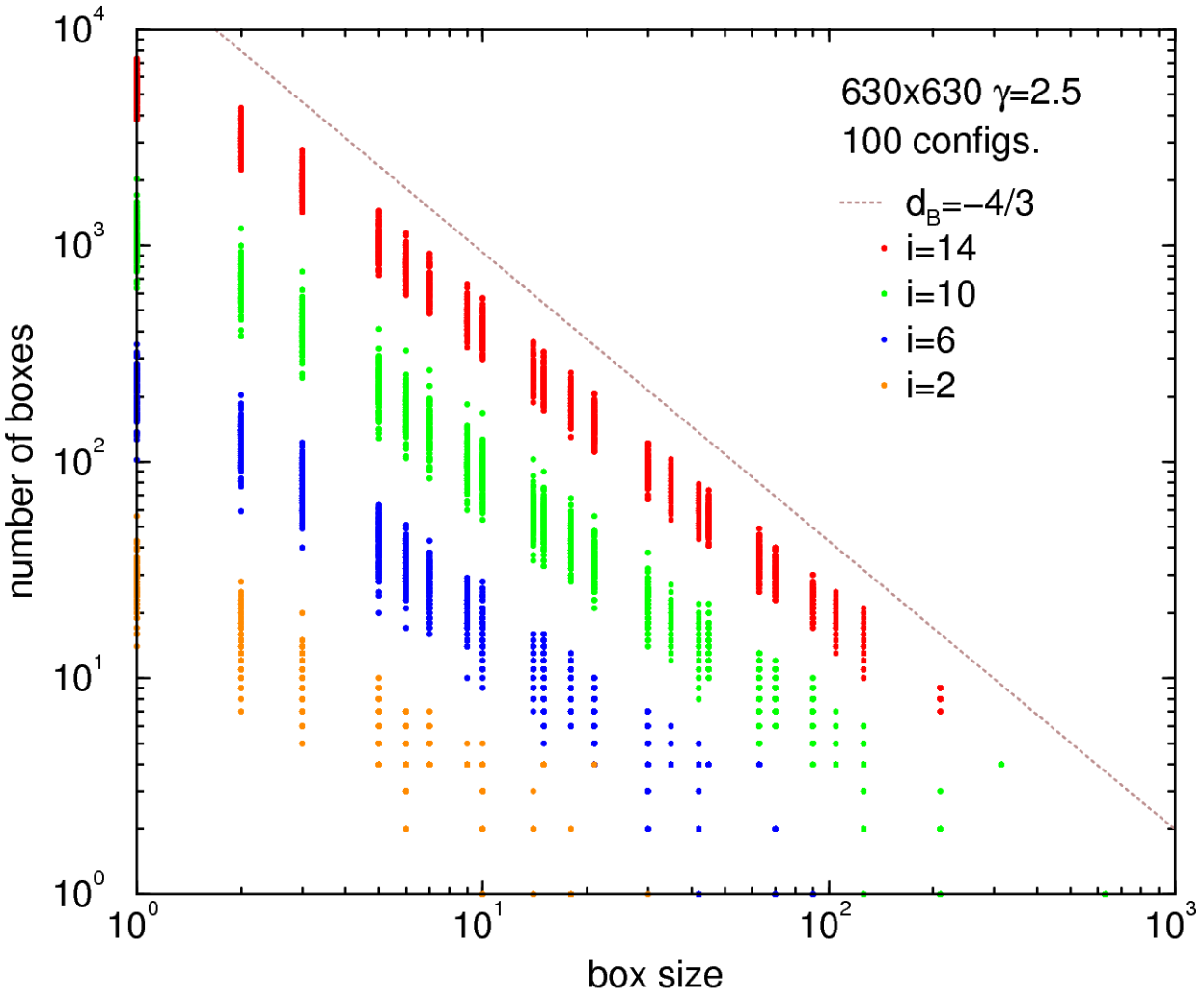
1 configuration

largest component

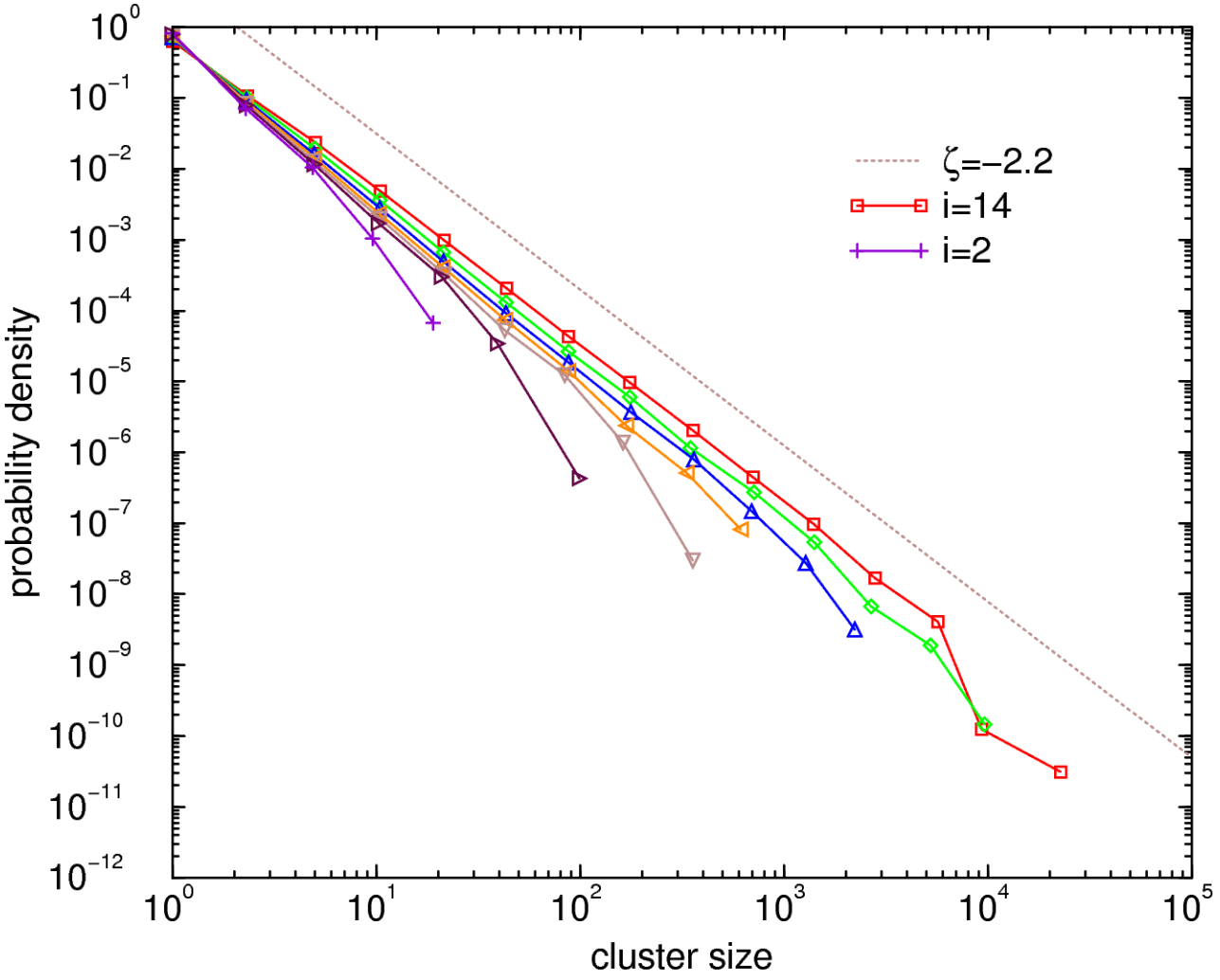
boundary

disregarding inclusions

Fractal dimension of boundary: box covering



Cluster size distribution (without largest)

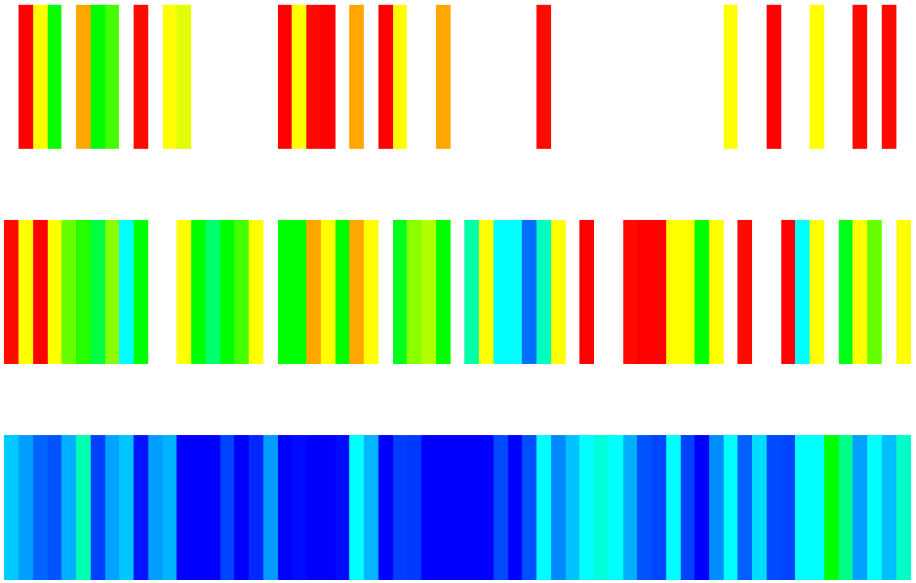


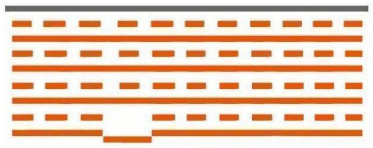
Summary

Simple & intuitive approach
Fractal boundary
Zipf's law

more work necessary
for characterization

Thank you for your attention.





http://diego.rybski.de/

http://www.pik-potsdam.de/members/rybski/