Company networks and their correlations beyond nearest neighbors

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PIK
Potsdam Institute for Climate Impact Research

Outline

1. Network fluctuation analysis (kFA)
2. Data
3. Company networks
4. Spatial embedding

1. Network fluctuation analysis (kFA)

Motivation

Degree correlations

- likelihood that nodes of given degree are connected
- assortative/disassortative mixing:
  positive/negative correlations
- measures:
  - Pearson correlation coefficient
  - average nearest neighbor degree
  - conditional probability $p(k_1, k_2)$
  - ...

disassortativity tightly related to *fractality* of complex networks

Average nearest neighbor degree

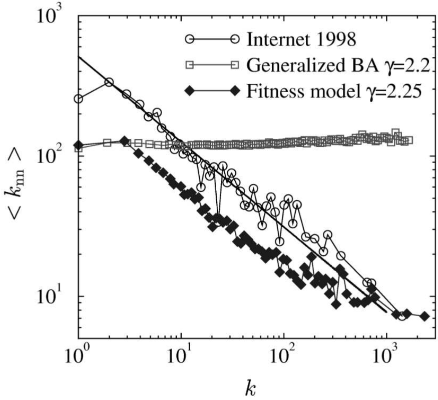

FIG. 2. The average connectivity $\langle k_{nn} \rangle$ of the nearest neighbors of a node depending on its connectivity k for the 1998 snapshot of the Internet, the generalized BA model with $\gamma = 2.2$ (Ref. [8]), and the fitness model (Ref. [18]). The solid line has a slope -0.5. The scattered results for very large k are due to statistical fluctuations.

Figure: R. Pastor-Satorras et al. PRL 2001.

Conditional probability $p(k_1, k_2)$

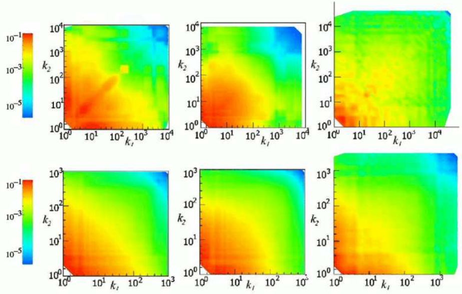

FIG. 1 (color online). The joint degree distribution $P(k_1, k_2)$ of WWW (top row) and Internet at the router level (bottom row) before renormalization (left), after renormalization forbidding multiple links (center), and including multiple links (right).

Figure: L.K. Gallos et al. PRL 2008.

Motivation

But:

- only correlations between nearest neighbor nodes, i.e. distance 1
- much of the rich topological information gets lost
- **how can correlations be measured at larger distances?**

Fluctuation Analysis

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4. Find the shortest paths between all pairs of nodes and determine the corresponding averages, $K_{ij} \ \forall \ i \neq j$.
5. Calculate the fluctuation function $F(d) = \sigma(K_{ij}|d)$, the conditional standard deviation of the K_{ij} at distance d.

In analogy to Fluctuation Analysis in time series analysis

Long-range (anti-) correlations

If the covariance, $C(d) \sim \langle (k_i - \langle k \rangle)(k_j - \langle k \rangle) | d \rangle$, between the degrees at distance d scales as

$C(d) \sim d^{-\gamma}$	for positive correlations (assortative) or
$C(d) \sim -(d^{-\gamma})$	for negative correlations (disassortative)

then we expect

$$F(d) \sim d^{\alpha_k} ,$$

where $\alpha_k = -\gamma/2$.

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$\alpha_k > -1/2$	positive correlations (assortative)
$\alpha_k = -1/2$	uncorrelated
$\alpha_k < -1/2$	negative correlations (disassortative)

Long-range (anti-) correlations

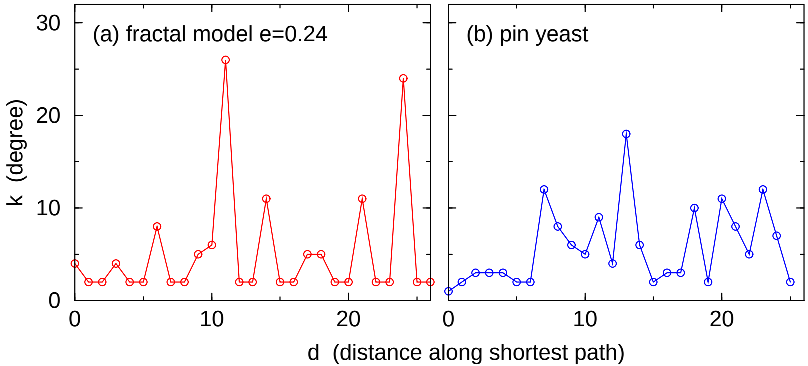

Figure: Degrees of nodes versus distance along shortest paths for (a) fractal network model and (b) pin yeast network.

1. Barabási-Albert model

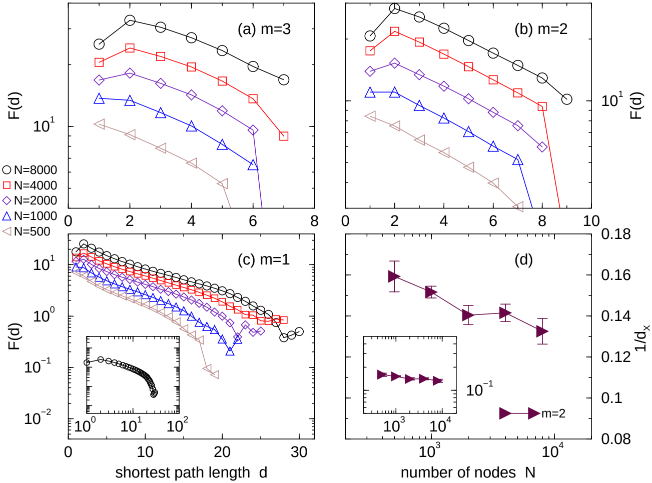

Figure: Degree fluctuation functions for the BA model. (a-c) show $F(d)$, (d) shows for $m = 2$ the slopes of exponential fits as a function of the network sizes. 100 configurations.

2. Cayley tree at percolation transition

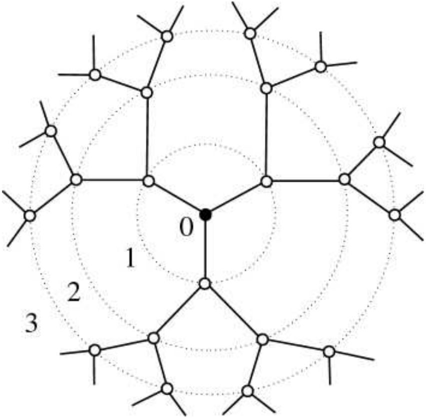

Figure: $z = 3$ (source: wikipedia)

percolation transition: $p_c = \frac{1}{z-1}$

topological dimension of giant component: $d_f = 2$

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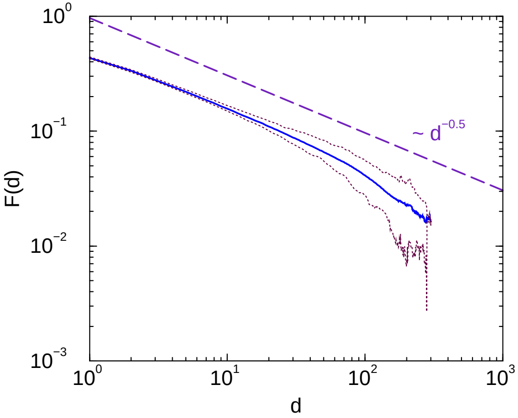


Figure: Degree fluctuation function of the Cayley tree at percolation transition ($z = 3$, $n = 150$). Dotted maroon lines: quantiles enclosing 90% (100 configurations).

3. Fractal network model

C.M. Song, S. Havlin, and H.A. Makse, nature physics, 2006.

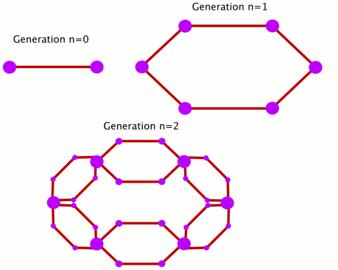


Fig. 1. Construction of a pure fractal network. Example of network model with parameters $n = 0, 1, 2; m = 2; x = 2; e = 0$.

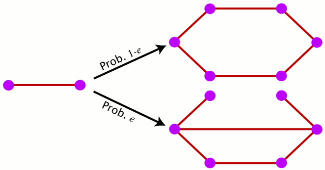


Fig. 2. Construction of network. With probability e the link between hub remains, otherwise, with probability $1 - e$ it is replaced for another link between new nodes.

Figure: generation n, m new nodes, x new links, probability e (source: H.D. Rozenfeld and H.A. Makse, 2009)

fractal dimension: $d_{\mathrm{f}} = \frac{\ln(2m+x)}{\ln(3-2e)}$

degree distribution: $p(k) \sim k^{-\left(1+\frac{\ln(2m+x)}{\ln m}\right)}$

3. Fractal network model

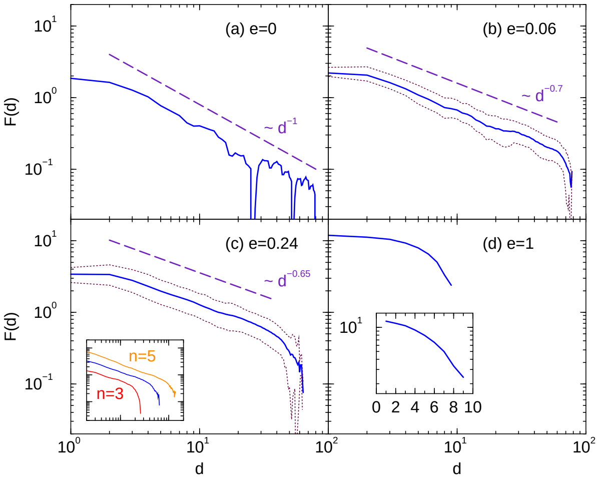


Figure: Degree fluctuation functions for the fractal model ($n = 4$). Dotted maroon lines: quantiles enclosing 90% (250 configurations). Inset in (c): $n = 3$ and $n = 5$ (25 configurations).

3. Fractal network model

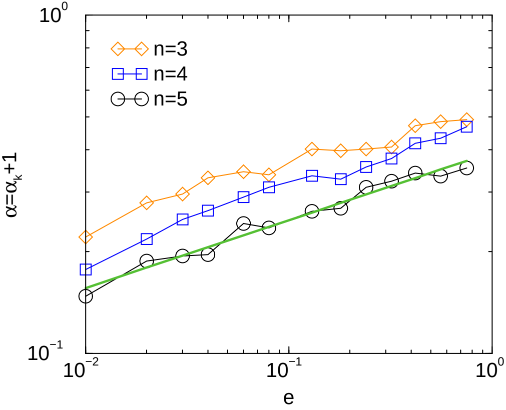


Figure: Exponents $\alpha = \alpha_k + 1$ of the fractal network model. Power-law fit for $n = 5$: $\alpha \sim e^{\epsilon}$, $\epsilon \approx 0.2$.

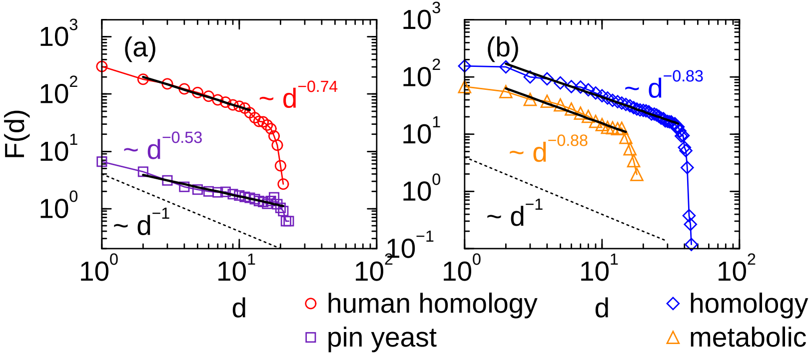
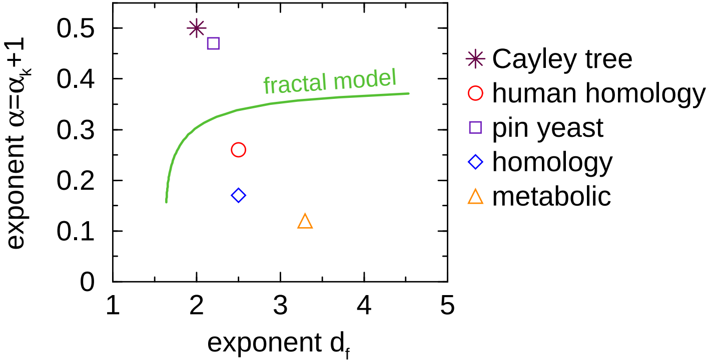


Figure: Degree fluctuation functions for real-world networks.

Comparison with fractal dimension



network	*k***FA**	**box covering** (Song et al.)
human homology	$\alpha_k \simeq -0.74$	$d_\mathrm{f} \simeq 2.5$
pin yeast	$\alpha_k \simeq -0.53$	$d_\mathrm{f} \simeq 2.2$
homology	$\alpha_k \simeq -0.83$	$d_\mathrm{f} \simeq 2.5$
metabolic	$\alpha_k \simeq -0.88$	$d_\mathrm{f} \simeq 3.3$

Findings

- BA model: exponential decay
- Cayley tree: $\alpha_k = -1/2$ (uncorrelated)
- fractal network model:
  - $e = 0$: $\alpha_k = -1$ (long-range anti-correlated)
  - $e = 1$: exponential decay
- real-world networks: power-law decay
- fluctuation exponent complementary information to fractal dimension

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- BA model: exponential decay
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- vary parameters m and x
- analytical description
- also other spatial correlations:
  - various network properties (clustering, betweenness, ...)
  - additional information available (time of addition, activity, ...)

2. Data

Data

Motivation

In economic geography it is believed that economic activity is influenced by

- space, i.e. spatial distance
- network, i.e. network distance
- topic, i.e. distance in terms of content

Ergo:
small research fields via defined research topics
collaboration networks via co-authorship

Data

Data collection (*Hennemann S. et al., submitted 2011*)

publications from isi web of science (2004-2008)
network of institutions with at least 1 common publication

keyword	**nodes**	**links**	**diameter**
bluetooth	439	515	13
image-compression	599	690	12
heart-valve	835	1589	17
tissue-engineering	2505	7443	10

keyword	**universities**	**res. inst.**	**companies**	**hospitals**
bluetooth	0.57	0.13	0.27	0.03
image-compression	0.69	0.13	0.14	0.04
heart-valve	0.42	0.18	0.07	0.34
tissue-engineering	0.44	0.19	0.17	0.20

further information: countries, geo-tagging

3. Company networks

Measuring long-range correlations

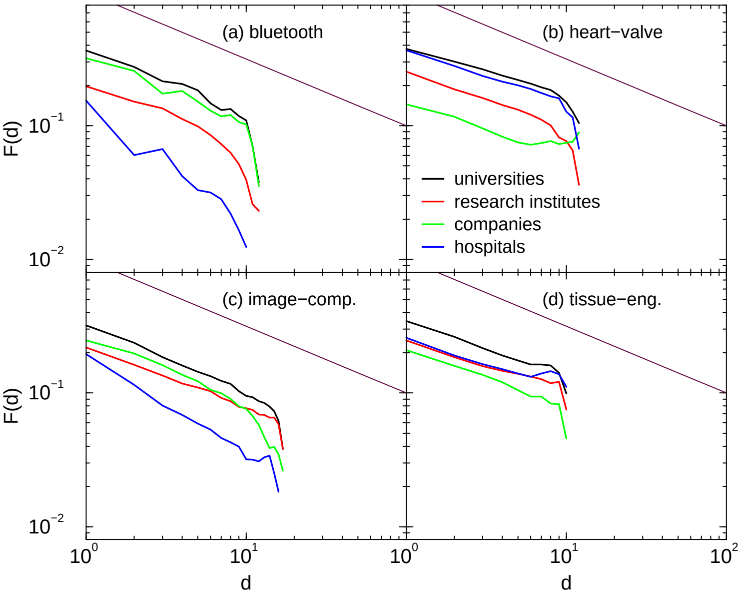
Applying fluctuation analysis

("fluctuation" in space – not in time)

- each node is associated to 1 of 4 types
  (universities, research institutes, companies, hospitals)
- pick one type
- replace the attribute of each node with
  - "1" if type matches
  - "0" if it does not match
- network fluctuation analysis on the 0/1-nodes attributes
- repeat for all 4 types

correlation patterns are reflected in fluctuation functions

Results: fluctuation functions



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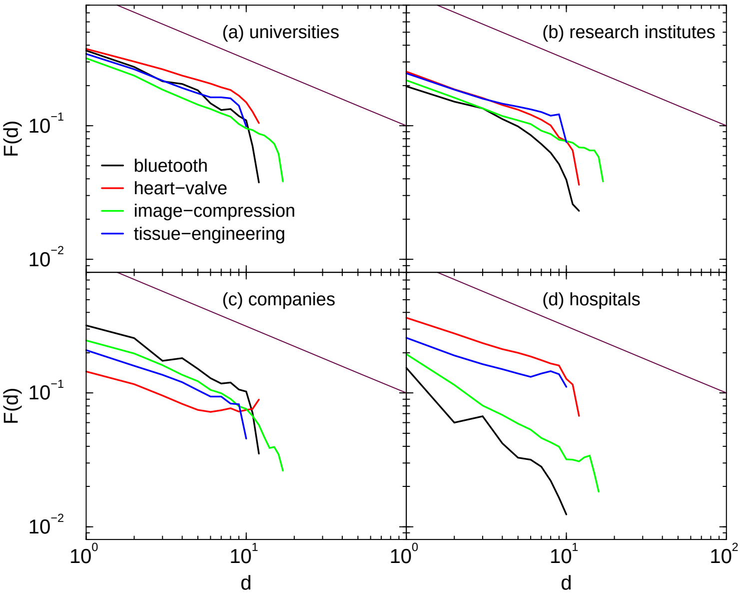
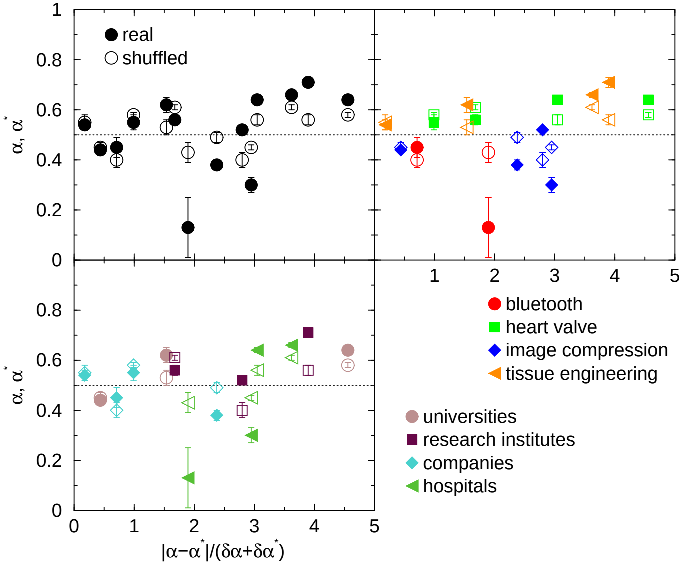
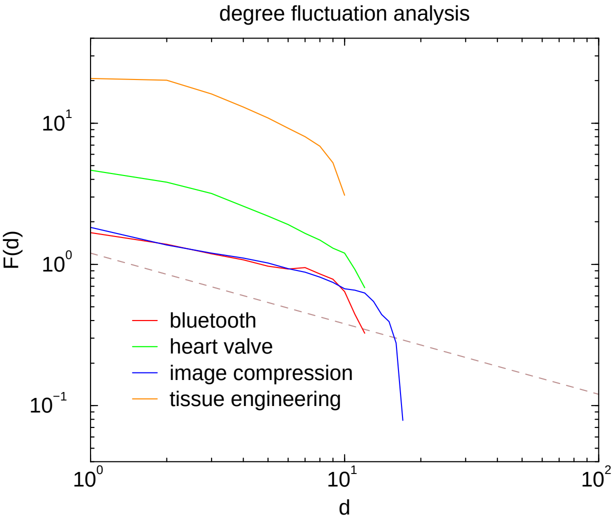


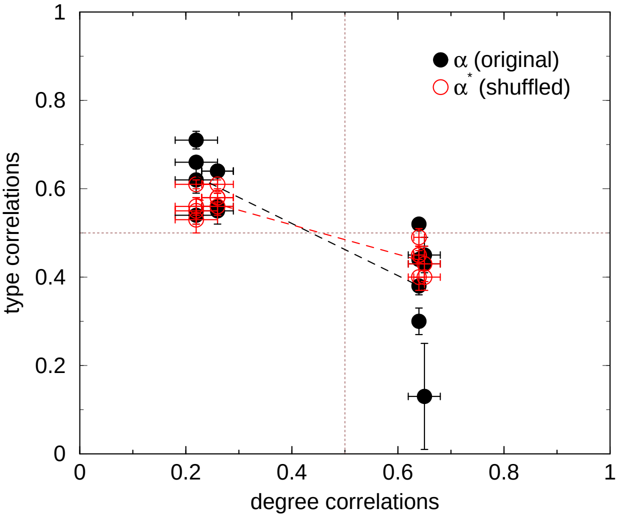
Results: fluctuation exponents



Results: degree fluctuation functions



Results: exponent cross-plot



Findings

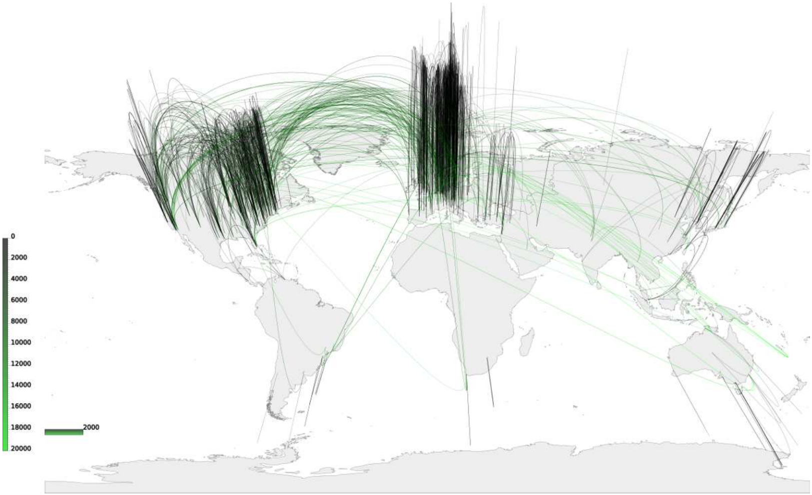
- scaling in the spatial fluctuations of institutions
- both, positive and negative correlations
- strength seems to depend on the network (i.e. topic)
- shuffling reduces correlations
- also correlations in the degree
- fluctuation exponents of type and degree seem to be anti-correlated

4. Spatial embedding

Motivation

- role of *proximity* is discussed in geography, economics, and innovation research
- faster and cheaper communication and traveling
- "shrinking world"
- to which extent reflected in company networks, i.e. collaboration networks?

map: heart-valve



example: tissue engineering

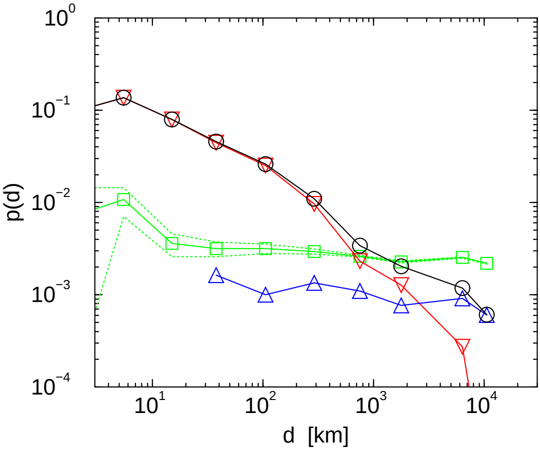
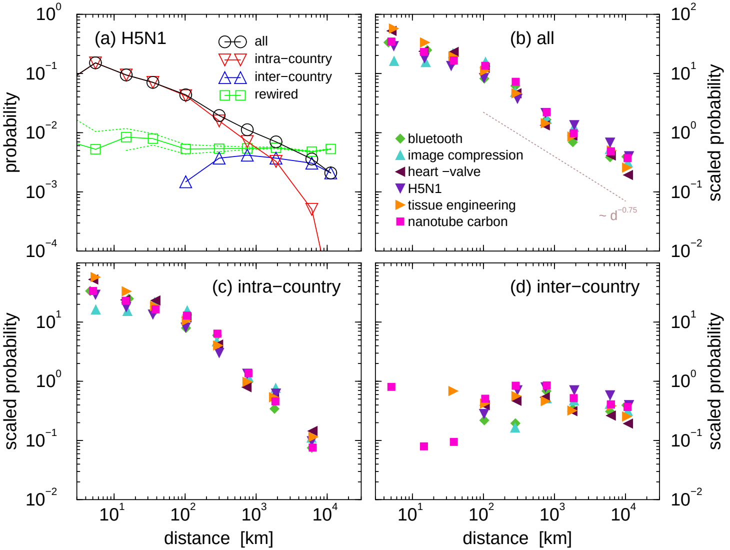


Figure: probability to have a link at distance d – not distribution of distances

collapse


(a) H5N1
all
intra−country
inter−country
rewired
probability
(b) all
bluetooth
image compression
heart −valve
H5N1
tissue engineering
nanotube carbon
~ d^−0.75
scaled probability
(c) intra−country
(d) inter−country
distance [km]
distance [km]

Thank you for your attention!

Acknowledgment:

- Organizers of the conference
- German Academic Exchange Service (DAAD)

Publications:

- Long-range degree-correlations: Rybski D. et al. EPL 2010
- Company networks: Rybski D. et al. (in prep.)
- Myth of global science collaboration: Hennemann S. et al. (submitted 2011)

http://diego.rybski.de/