POTSDAM INSTITUTE FOR CLIMATE IMPACT RESEARCH

New Laws of City Growth

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FU Berlin,
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The City College of New York

How to define cities?

Different definitions of cities affect statistical properties of urban activity.

Method to define cities by the US census bureau: MSAs, places, counties, etc.

Conventional Method to define (large) “cities” in the USA: Metropolitan Statistical Areas (MSA).

MSAs are subject to socio-economical factors and constructed manually, done only for the largest cities (about 300) and cannot be easily applied in other countries.

Outline

I. City Clustering Algorithm (CCA)

II. City size (Zipf's Law)

III. City growth (Gibrat's Law)

City growth (Gibrat's Law) & CCA:
Rozenfeld HD, et al. (2008) PNAS 105:18702-18707.

City size (Zipf's Law):
Rozenfeld HD, et al. (2011) AER accepted.

I. City Clustering Algorithm (CCA)

A new definition of cities

We define a new way to construct cities:

- unbiased
- automated
- fast
- can be easily used in any country
- based only on location of population
- allows studying cities at different level of observation

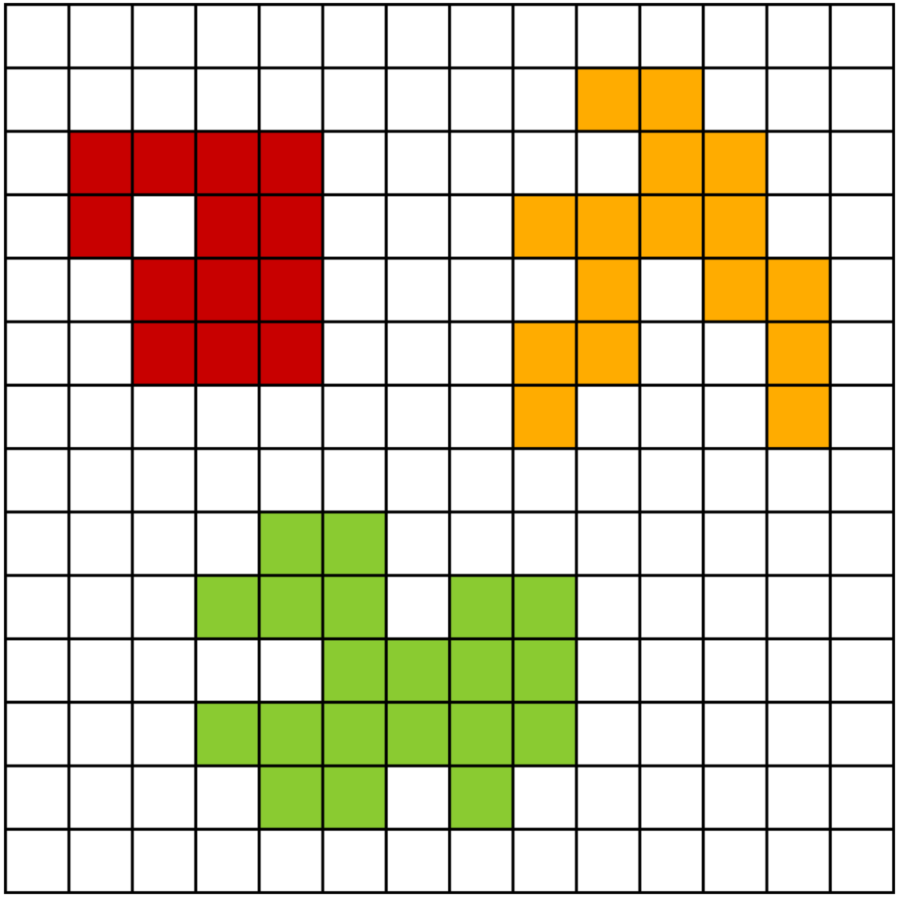
City Clustering Algorithm (CCA)

Data

Great Britain (England, Scotland, and Wales):
58.7 millions in 2007
0.23 million km2
grid of 200m x 200m

USA:
303 millions in 2007
7.44 million km2
59456 sites (FIPS)

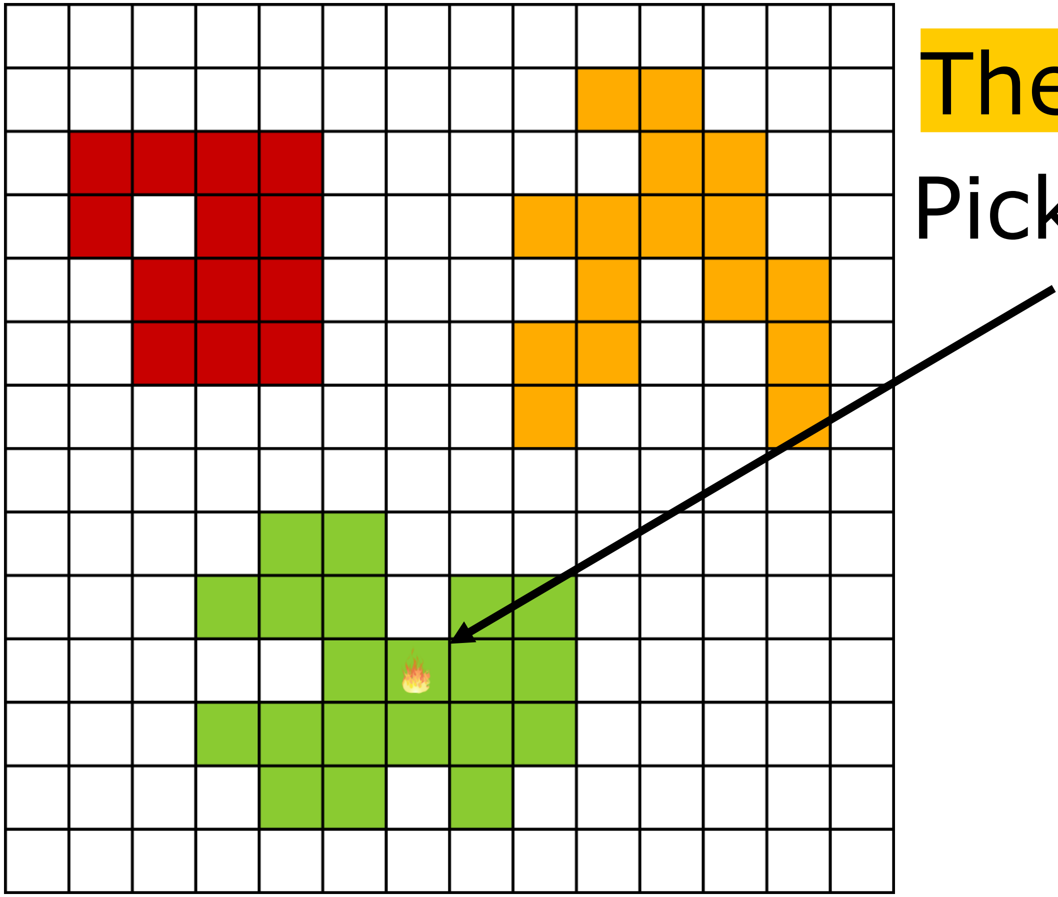
A new definition of cities



The map is gridded.
The populated cells are identified.

We define:
“a city” = “a cluster of connected populated cells, with maximal size”

A new definition of cities



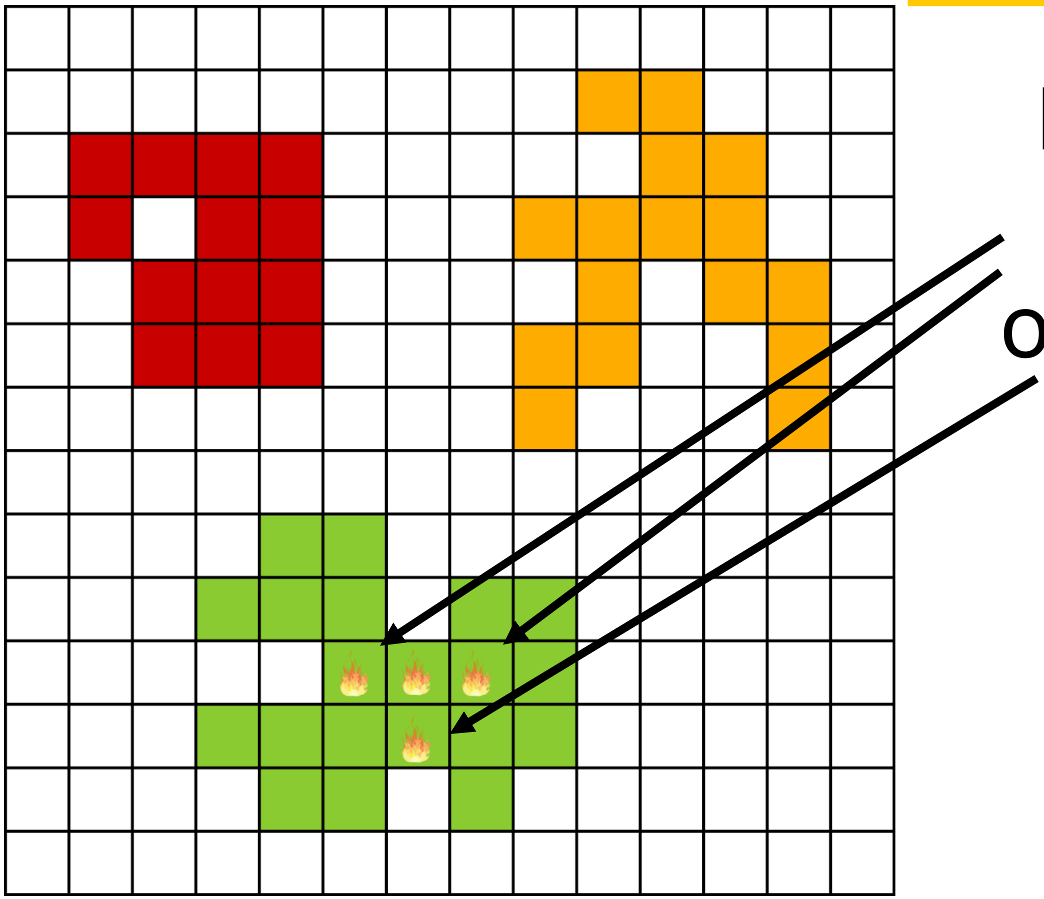
The burning algorithm.

Pick a populated cell and burn it.

Stauffer *Introduction to percolation theory* ('84).

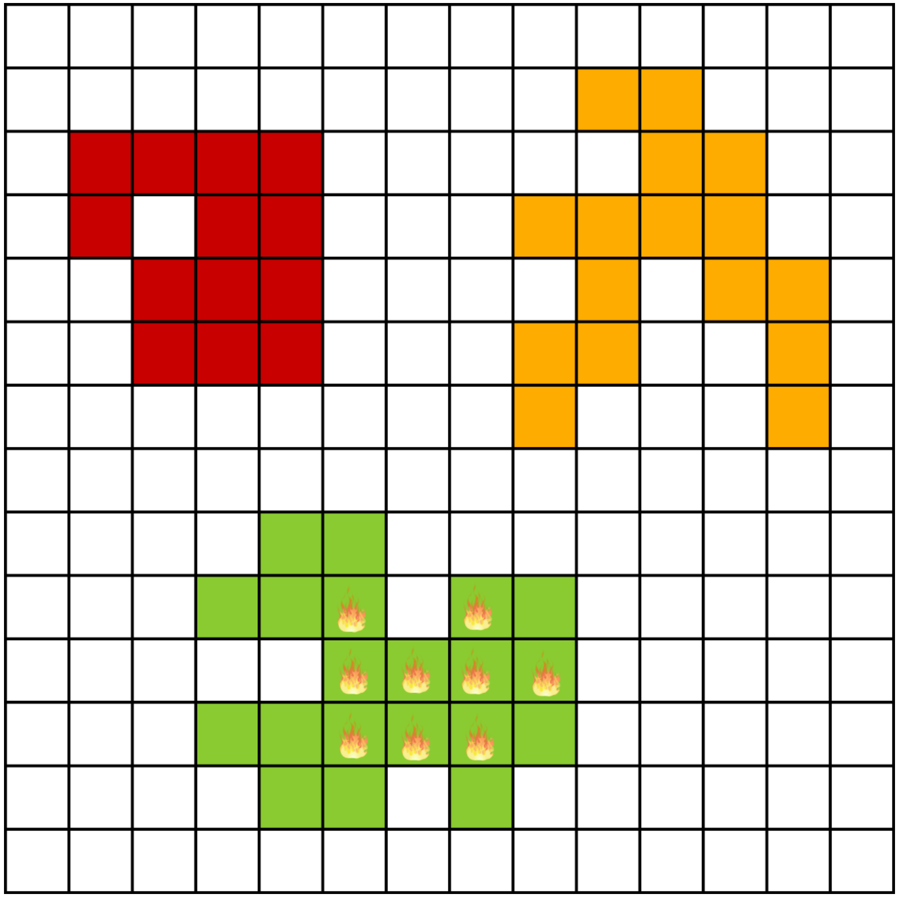
A new definition of cities

The burning algorithm.



Find the populated neighbors
of the burnt cell and burn them.

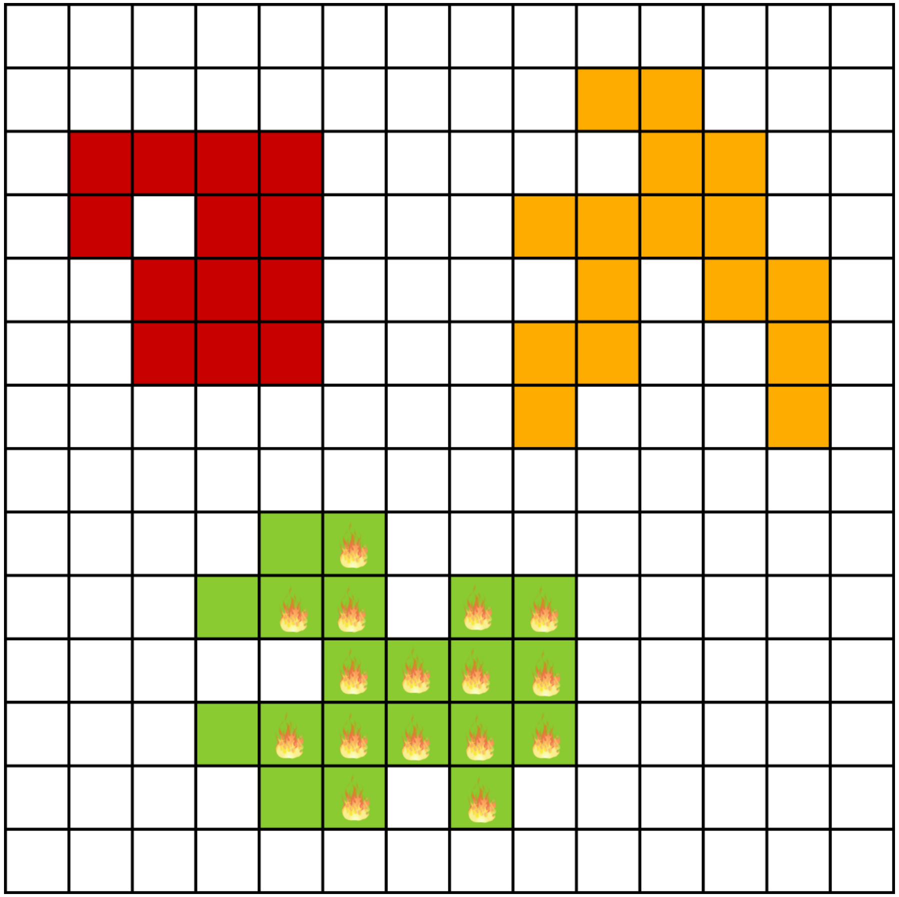
A new definition of cities



The burning algorithm.

Find the populated neighbors of the burnt cells and burn them.

A new definition of cities

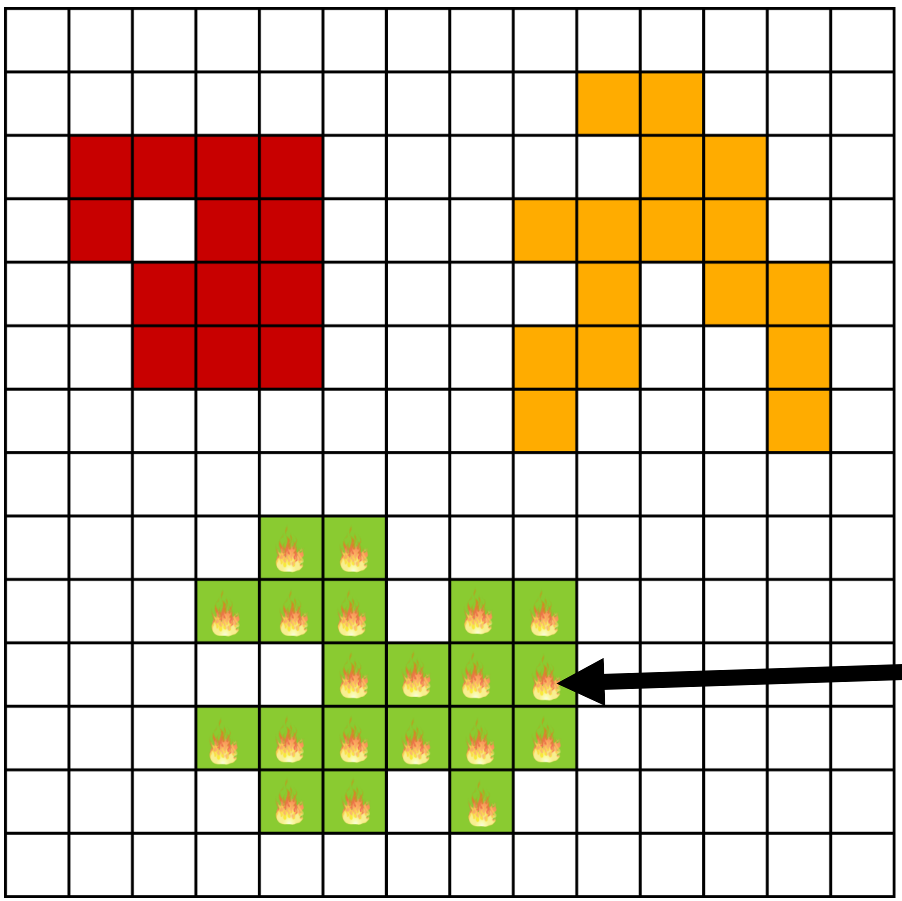


The burning algorithm.

Recursively,
continue identifying
the populated
neighbors
of the burnt cells
and burning them.

A new definition of cities

The burning algorithm.

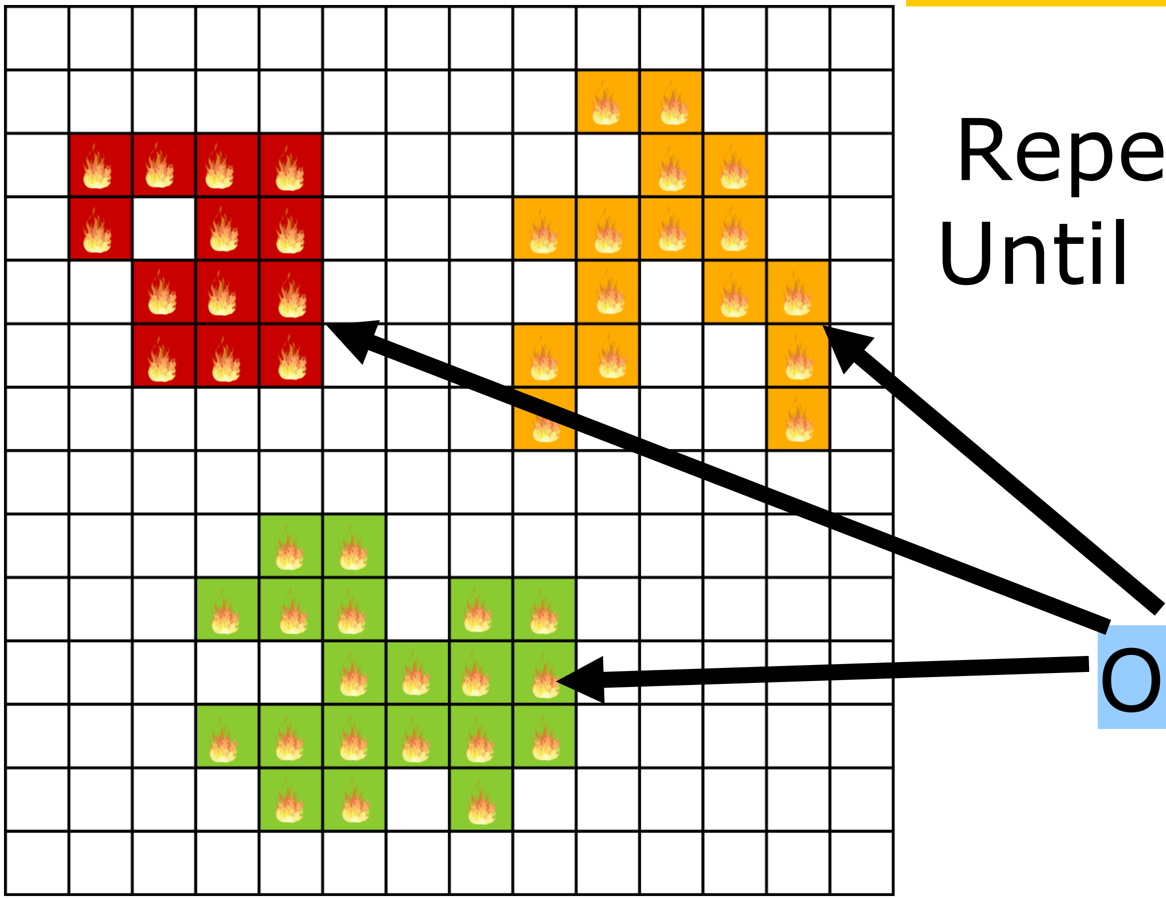


When all burnt cells have no populated neighboring cells, the cluster is completed.

Our first cluster!

A new definition of cities

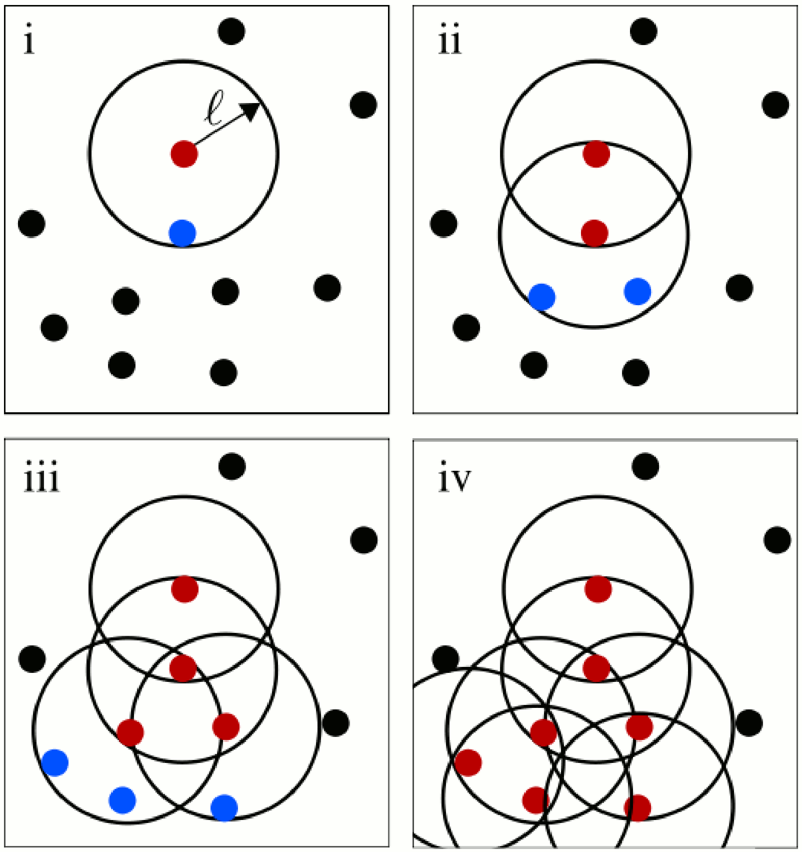
The burning algorithm.



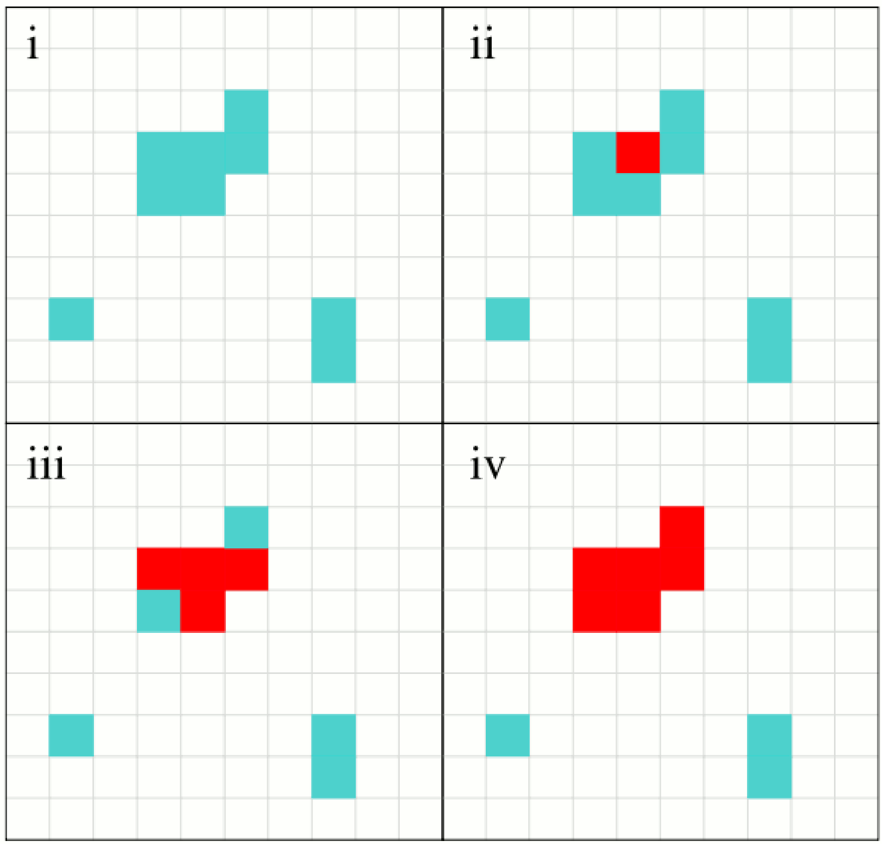
Repeat the procedure
Until all populated cells
are burnt.

Our three clusters!

A new definition of cities

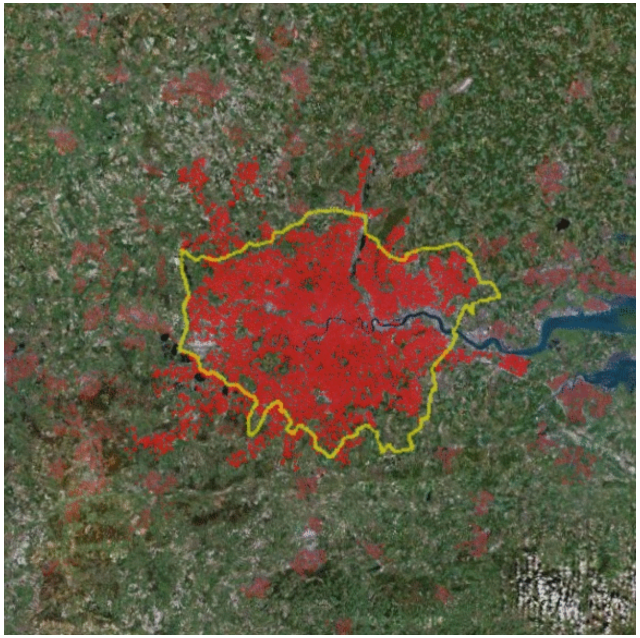


US

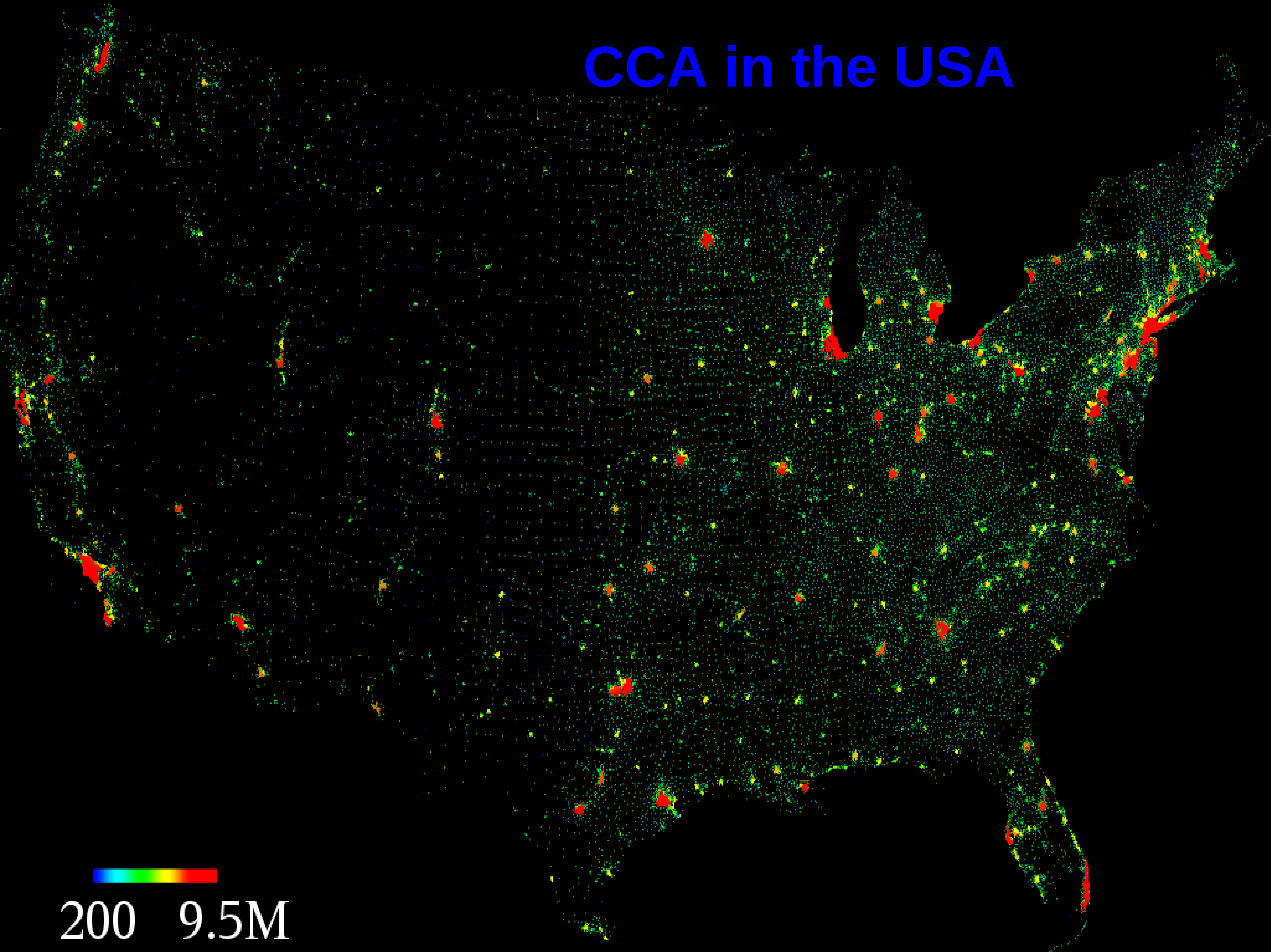


GB

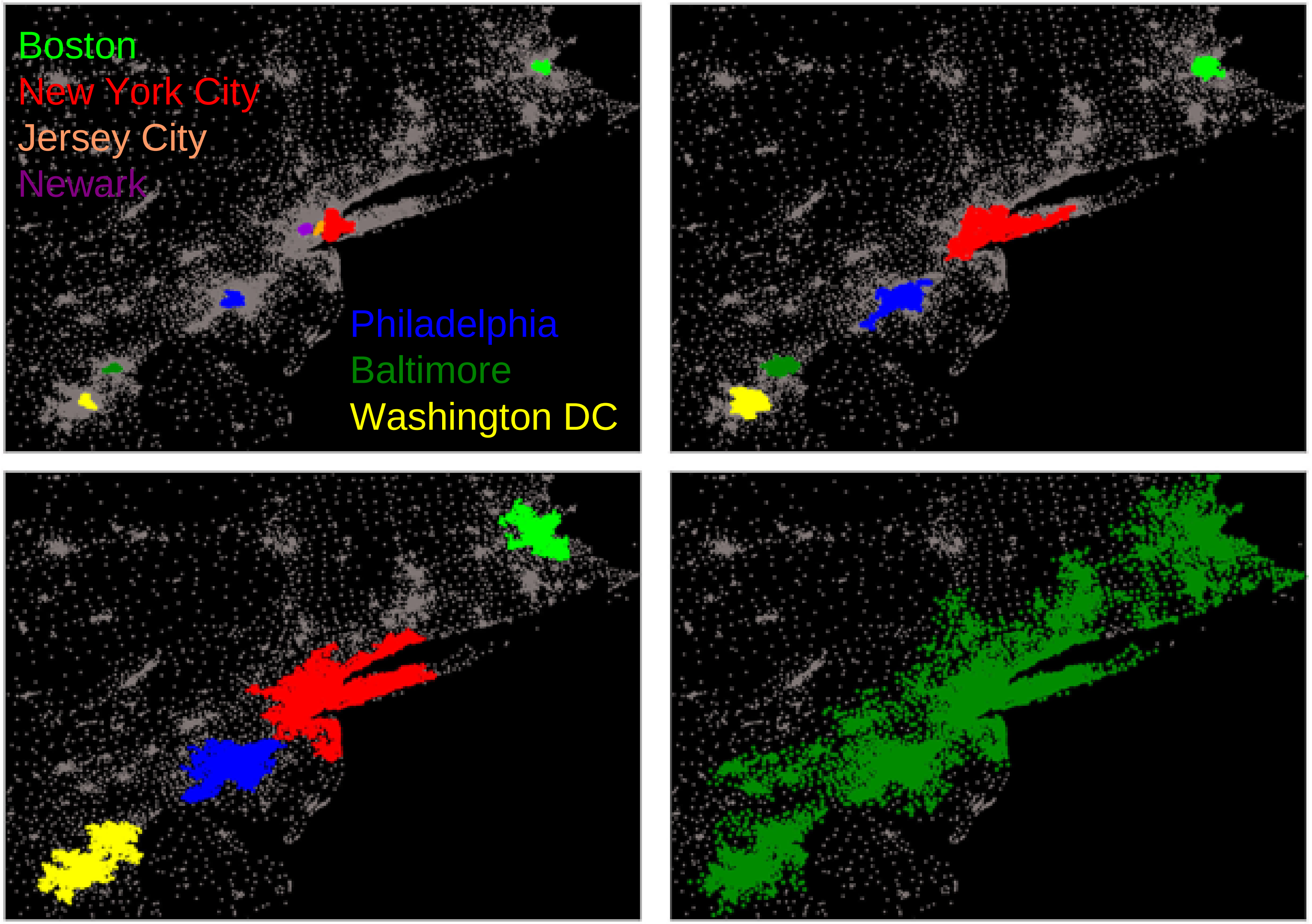
CCA in Great Britain



CCA applied
to Greater London


CCA in the USA
200
9.5M

CCA in the USA



II. City size (Zipf's Law)

Zipf's Law

$$P(S) \sim S^{-\zeta-1}, \quad \zeta = 1$$

The distribution of sizes follows
a power-law with $\zeta = 1$

Zipf's law has been documented for
words, firms, size of exports,
and many more

Does the city size distribution obey Zipf’s Law?

Zipf's Law

Understanding the origin of this regularity is an ongoing task.

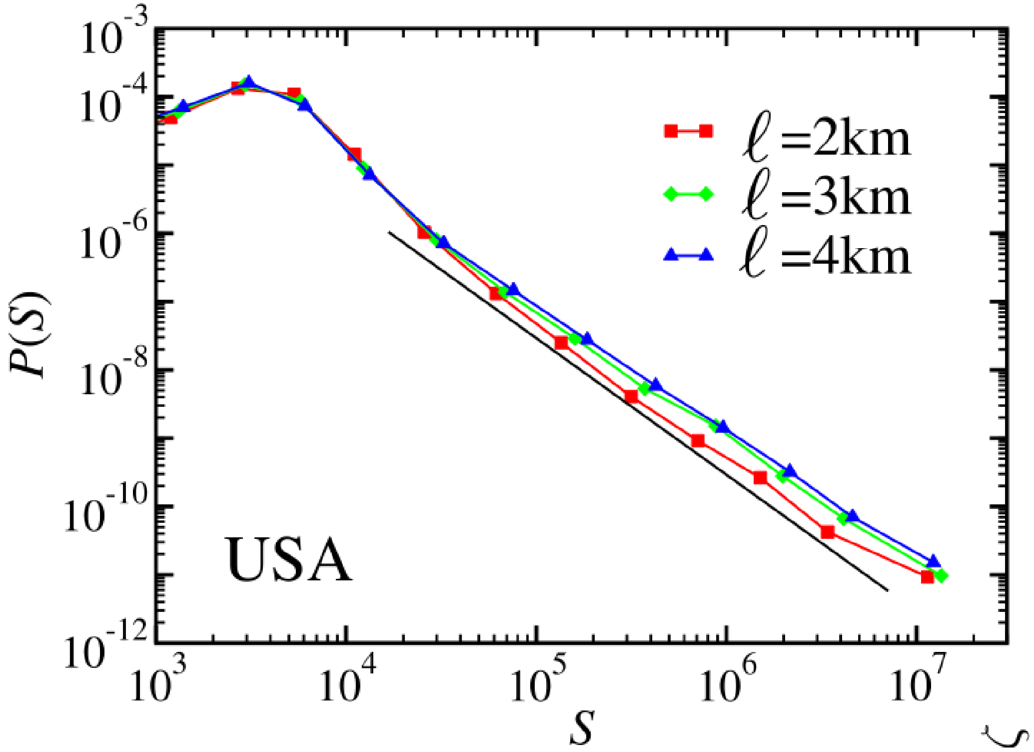
Typically, studies use MSAs for the top 200 cities, i.e. Eeckhout ('07)

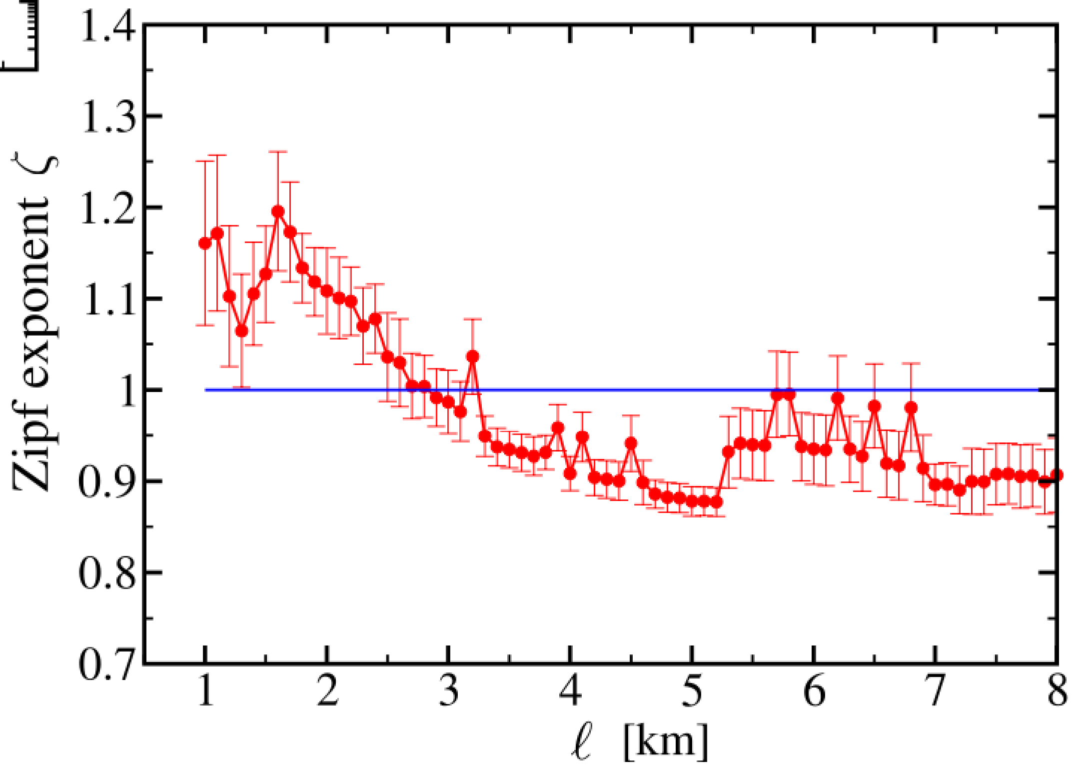
Eeckhout ('07)

- Uses data on all administrative cities
- Finds a very good log-normal fit

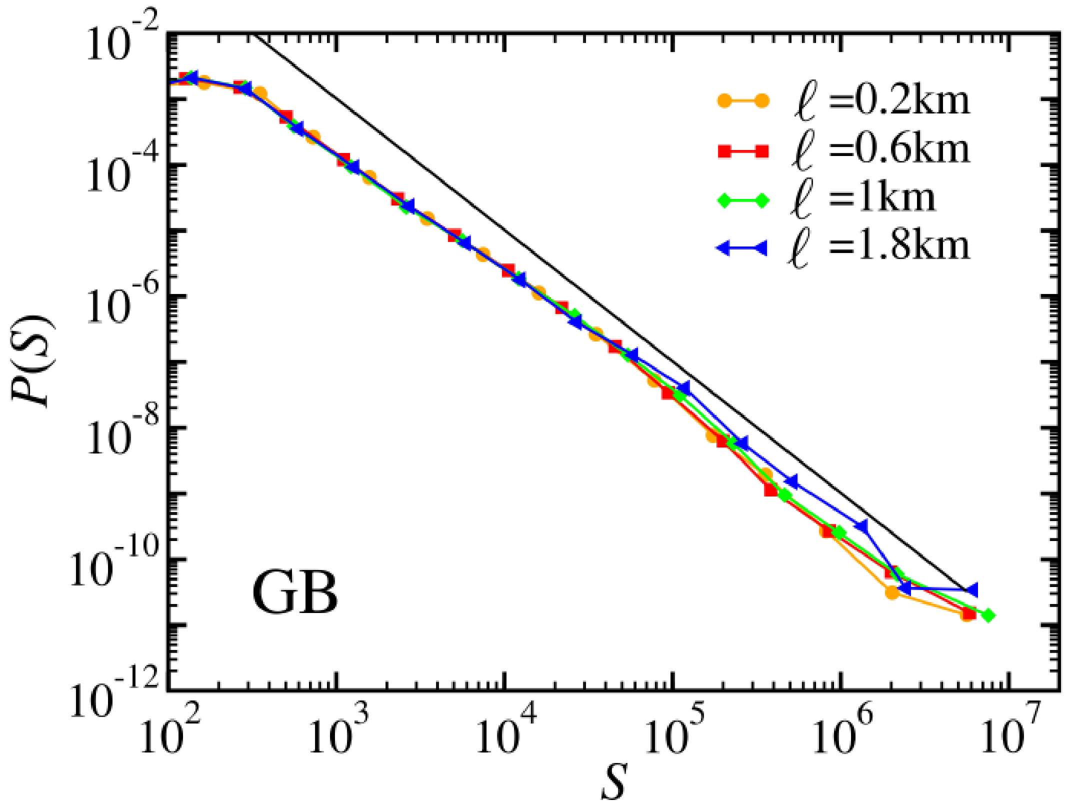
Distribution of city size using the CCA?

Zipf's Law for the USA

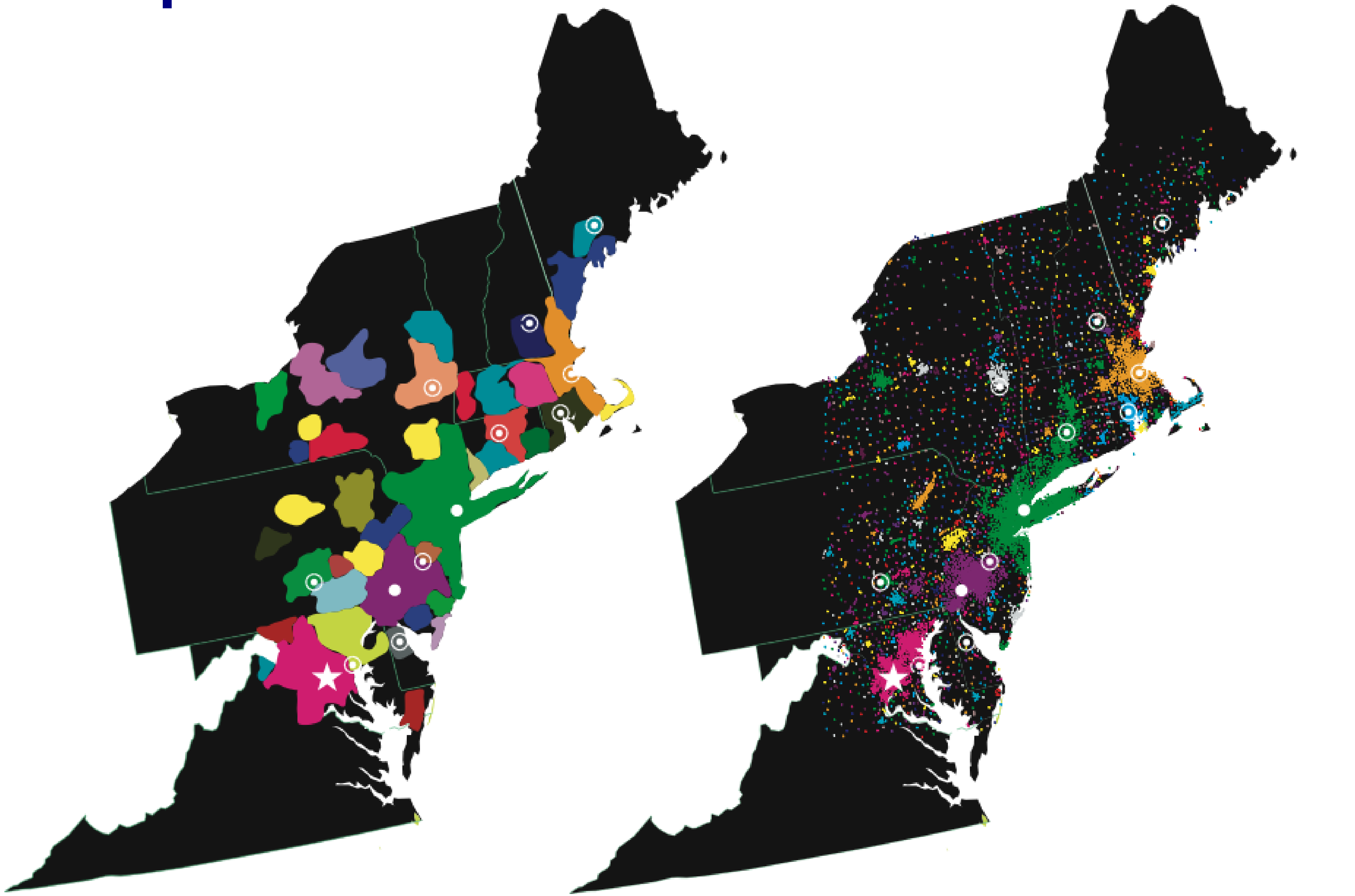




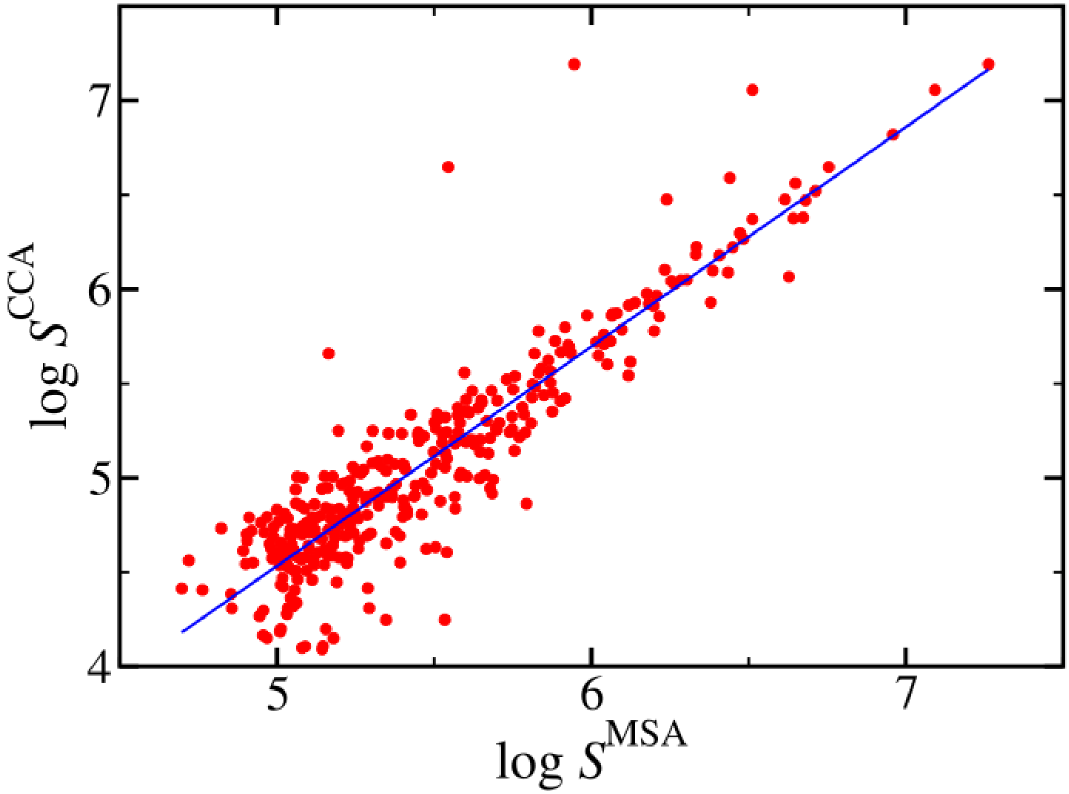
Zipf's Law for the GB

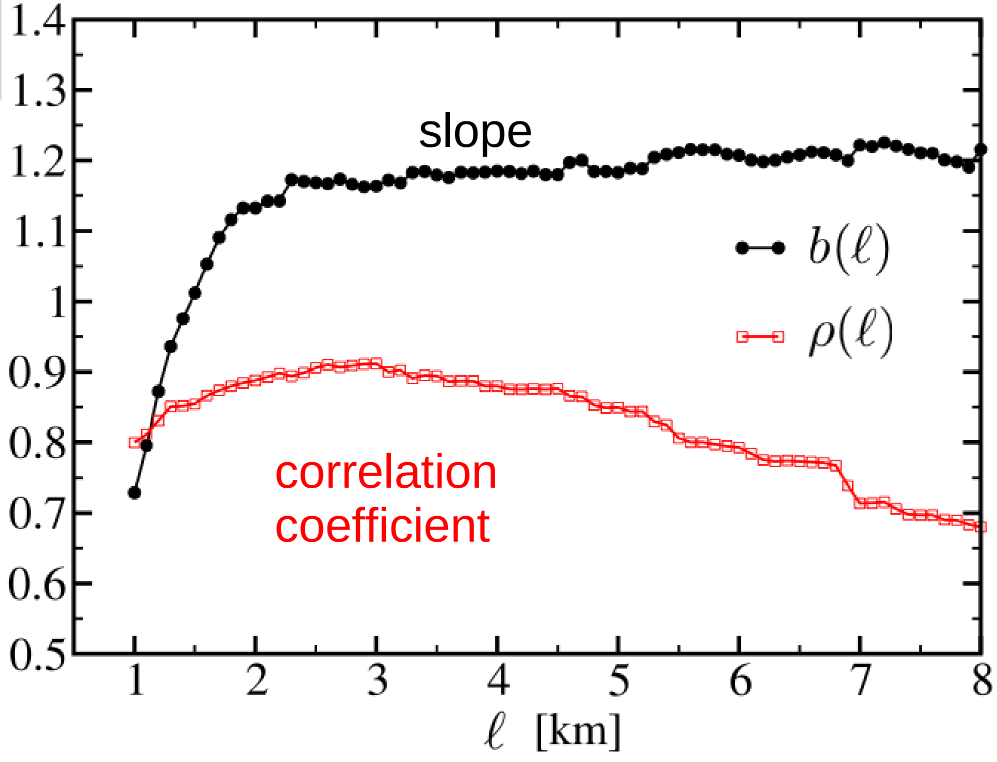


Comparison with MSA: Northeastern USA

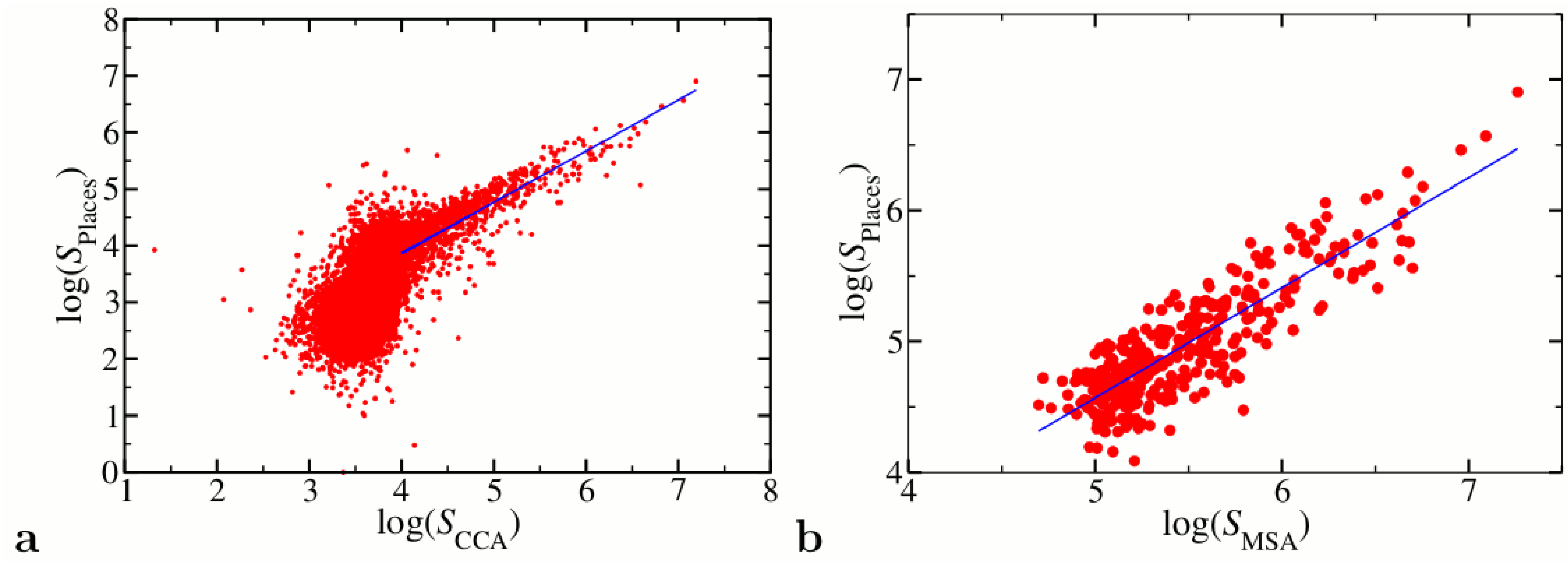


Correlations between MSA and CCA

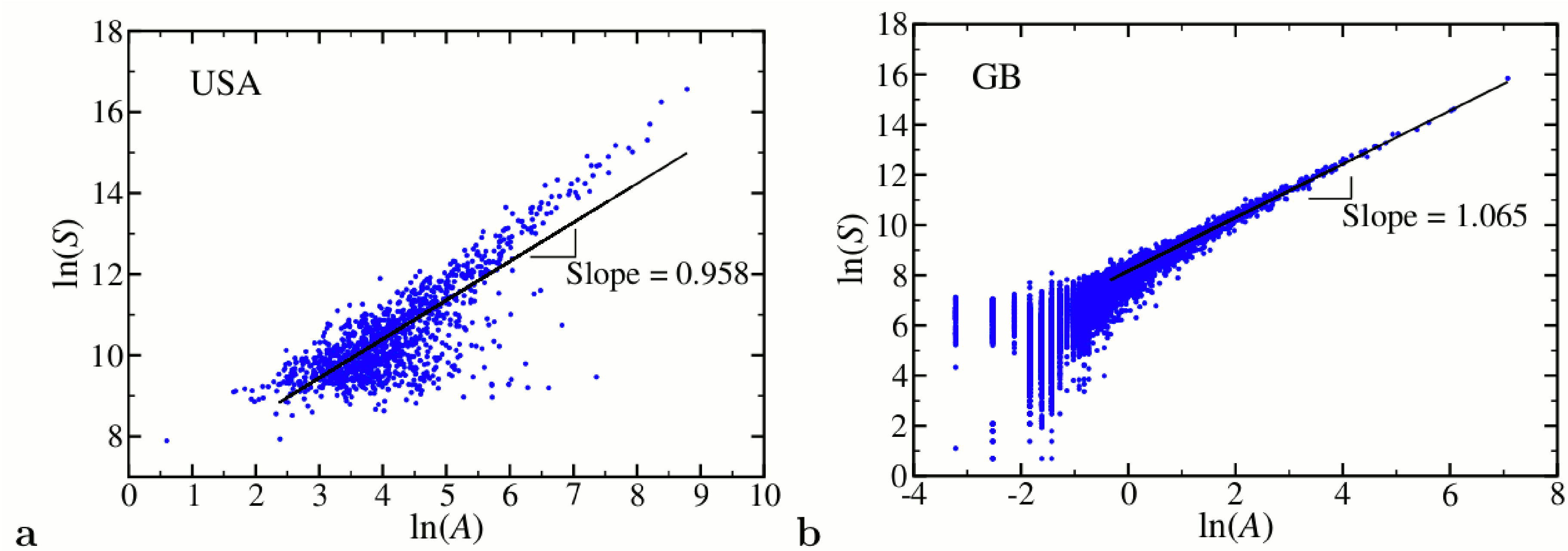




Correlations with Places



Correlations with Area



III. City growth (Gibrat's Law)

City growth

S_0 Population of a city at time 0.

S_1 Population of a city at time 1.

$$S_1 = R(S_0)S_0 \longrightarrow R \text{ growth factor}$$

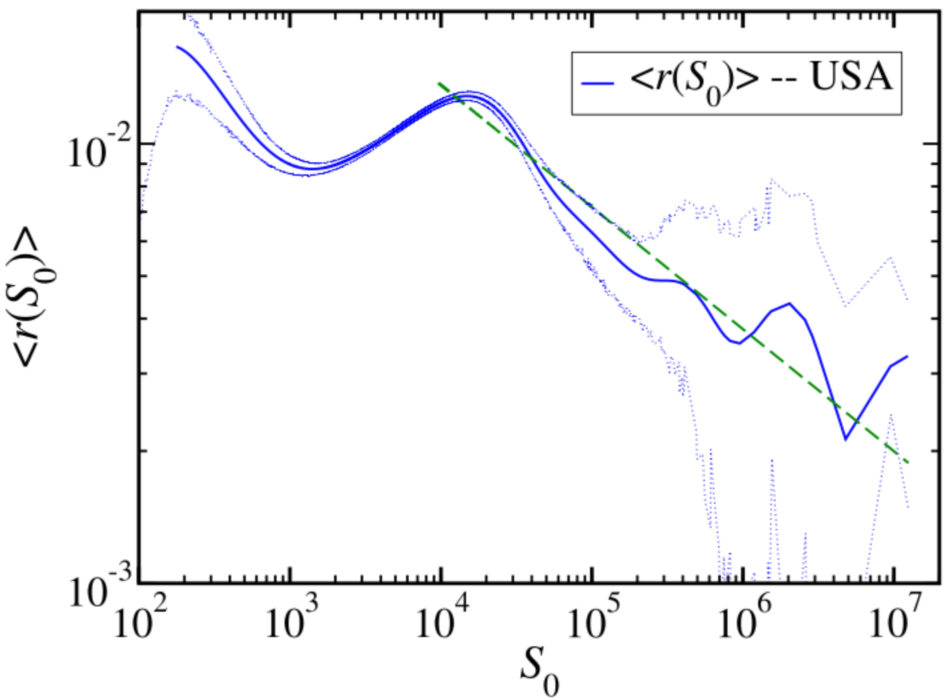
$$r(S_0) \equiv \ln R(S_0) = \ln(S_1/S_0) \longrightarrow r \text{ growth rate}$$

$$\langle r(S_0) \rangle \sim S_0^{-\alpha}$$

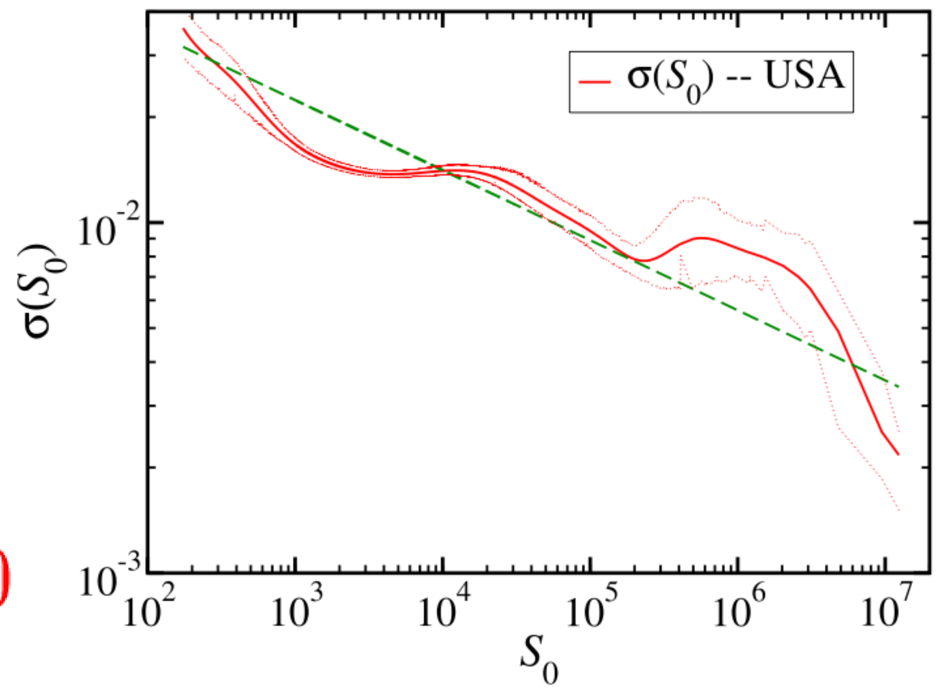
$$\sigma(S_0) = \sqrt{\langle r(S_0)^2 \rangle - \langle r(S_0) \rangle^2}$$

$$\sigma(S_0) \sim S_0^{-\beta}$$

City growth in the USA (1990-2000)



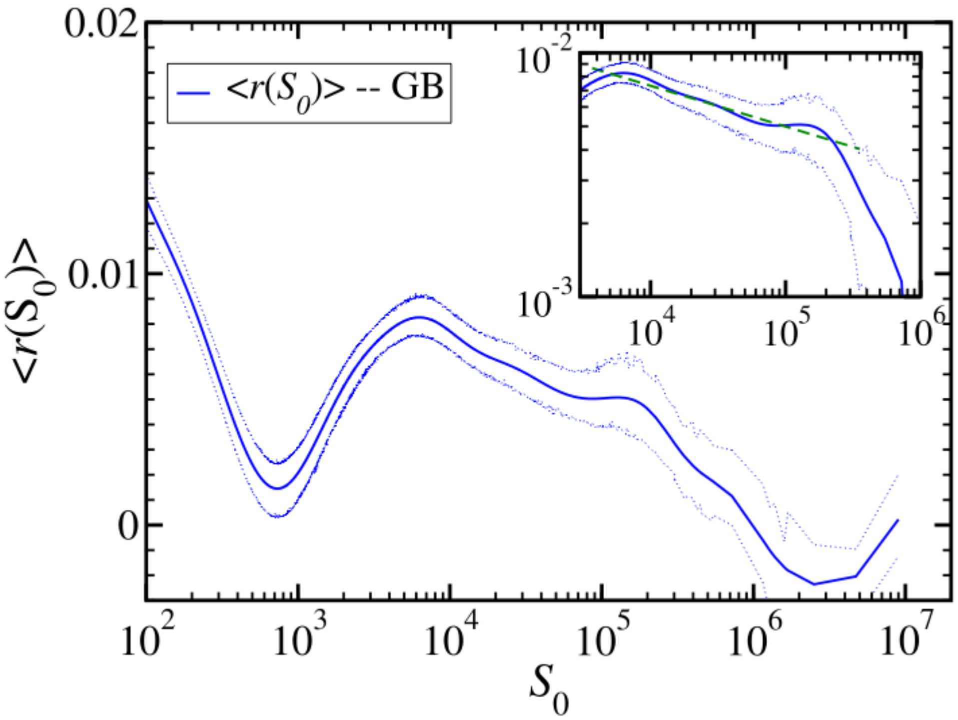
$$\langle r(S_0)\rangle \sim S_0^{-\alpha}\,, \quad \alpha = 0.28$$



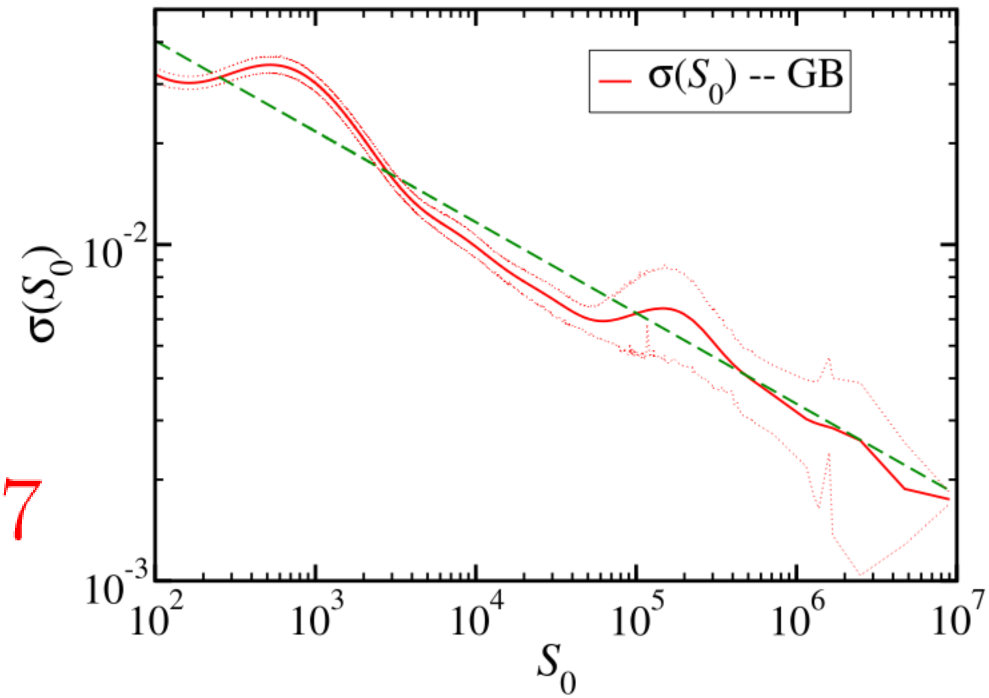
$$\sigma(S_0) \sim S_0^{-\beta}\,, \quad \beta = 0.20$$

Are not in agreement with Gibrat's Law (stating that average growth rate and standard deviation are constant)

City growth in the GB (1981-1991)



$$\langle r(S_0)\rangle \sim S_0^{-\alpha}\,, \quad \alpha = 0.17$$

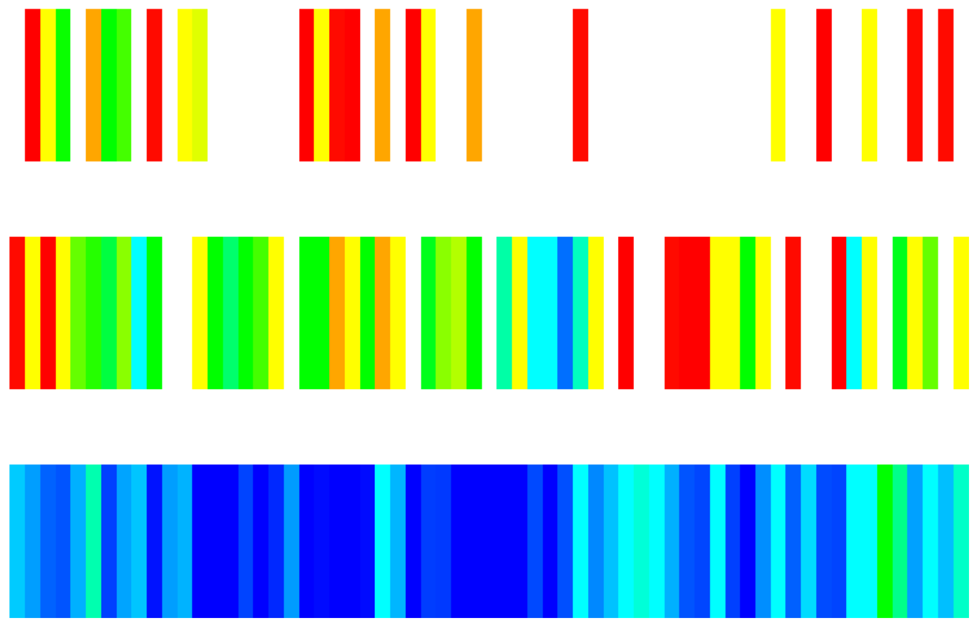


$$\sigma(S_0) \sim S_0^{-\beta}\,, \quad \beta = 0.27$$

Summary

- CCA constructs cities based only on geographical features
- Zipf's Law holds over a wide range, even for smaller cities
- Scale-invariant growth mechanisms at different geographical scales (violation of Gibrat's Law)
- Power-law standard deviation is due to long-range spatial correlations in the growth (not shown)
- How about other countries?

Thank you for your attention.



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